TRANSACTIONS
AMERICAN GEOPHYSICAL UNION

Volume 31 JUNE 1950 Number 3

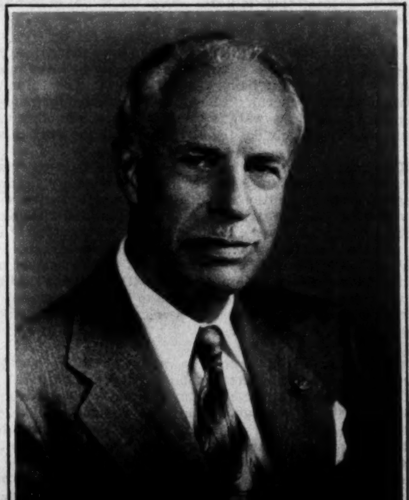
TWELFTH AWARD OF THE WILLIAM BOWIE MEDAL

CITATION

Presented by Merle A. Tuve

Mr. President: It may be recalled that the deed of gift of March 1, 1939 for the William Bowie Medal specifies that its award is to be for distinguished attainment and outstanding contribution to the advancement of cooperative research in fundamental geophysics.

In accordance with the unanimous action of the Union's Committee on the Award of the William Bowie Medal, I am privileged to recommend to the Union for the honor of the Twelfth Annual Award, a Geophysicist who not only meets in the fullest the requirements of attainment and contribution in the advancement of geophysics for over forty years, but is also an outstanding example of devotion to his chosen field. After several years as an Industrial Chemist, he took up work in physical chemistry with the U.S. Geological Survey. Before beginning, in 1910, his life service in research on the conditions of substances under high pressure including particularly the properties of minerals and rocks at high pressure, the production of optical glass and the internal constitution of the Earth, his contributions have involved not only geophysics but geology as well in which field he has a distinguished record.



Leason Heberling Adams
Recipient, Twelfth Award of the William Bowie Medal
May 1, 1950 (Photo by Harris and Ewing)

He became a member of our Union in 1920 and was President from 1944 to 1947. Presently, he has the distinction of being one of the two Vice Presidents of the International Union of Geodesy and Geophysics. Since 1938, as Director of the Geophysical Laboratory of the Carnegie Institution of Washington, he has ably added administration of that organization to his record of scientific achievement in the best tradition of selfless cooperation.

Mr. President, I present the name of one whose scholarly stature is of that pre-eminence in geophysics set by the eleven previous recipients of the William Bowie Medal, Dr. Leason Heberling Adams

PRESENTATION BY PRESIDENT WALTER H. BUCHER

The knowledge that is accumulated by the "Basic Sciences" is applied in two directions: To the past, when it leads to the understanding of how things came to be as they are; and to the future, when it tells how things can be changed and new things can be made. A century ago these three aspect of science were clearly recognized as "Natural Philosophy," "Natural History," and "Engineerin It is one of the great tasks of the American Geophysical Union to bring about mutual understanding of aims and effective cooperation between the men who look at the Earth with its envelopes as it is, and those who hold the keys to the understanding of how it came to be that way. In this, the American Geophysical Union has been remarkably successful because of the example set by the life work of men whose intellectual curiosity and endowments led them to be both "natural philosophers" and "natural historians." It is a great privilege to bestow upon you, Dr. Leason Heberling Adams, distinguished geophysicist and geologist, the William Bowie Medal, in the name of the American Geophysical Union.

RESPONSE AND ACCEPTANCE BY LEASON H. ADAMS

Mr. President: To receive this award is indeed a great honor. I had the privilege of knowing well and for a long time the great geophysicist for whom the Medal is named; and it thus has for me personal as well as scientific interest.

The William Bowie Medal is bestowed for cooperation in research. Although presented to an individual, it is to be remembered that cooperation necessarily involves more than one person; and I must conclude that actually what is being recognized on this occasion is the contribution to Earth science by the organization with which I have been connected for many years. Most certainly the scientific staff, past and present, as well as those colleagues in other groups with whom we have been so happily associated, largely deserve the credit for the work in which I have had a part.

The Geophysical Laboratory, like other departments of the Carnegie Institution of Washington, prides itself on collaboration in scientific research. That Laboratory, in carrying forward its long-range studies on the nature and composition of the globe on which we live, would have found progres difficult, if not impossible, had it not utilized to the utmost the joint thinking and the joint efforts of investigators with different training and divergent points of view. Specific areas of research, as well as comprehensive ones, profit by cooperation. For example, experimental petrology needs the wisdom of the field geologist in setting up the problems, the techniques devised by the physicist in order that pertinent quantities may be measured with precision, and the knowledge of reactions between molecules that the chemist contributes to the interpretation of the results. In field work also, as for instance in seismology and in volcanology, scientists can seldom work singl but rather must share responsibility and labor.

With what I hope may be considered pardonable vanity, we at the Geophysical Laboratory believe that we have opened up new fields in geophysics and geochemistry. For our own benefit and that of our field of science, we have been in close association with institutions and individuals with similar interests. Consistently throughout the years we have endeavored to broaden the applicability of our results and to facilitate our researches by opening our doors to specially qualified visitors from this and other countries. Among the eminent scientists from abroad whose association with us as visiting investigators has added luster to the Laboratory are Niggli from Switzerlan Jaeger and Brouwer from Holland, Boeke from Germany, Tilley from England, Eskola and Sahama from Finland, and Kôzu and Tsuboi from Japan. The staff members with whom I have joined, formally and informally, in research are too numerous to list on this occasion, but among former members may be mentioned Day, Allen, Wright, White, Washington, Fenner, Roberts, Johnston, and Sosman. The achievements of these and others form a part of the imperishable record of science; and I now wish to acknowledge the part that all past and present staff members have played in what is being recognized tonight.

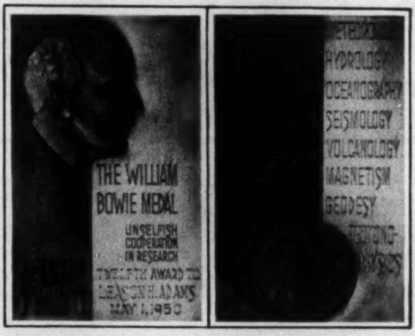
Mr. President, on behalf of those pleasant and gifted people with whom I have been happy to cooperate, I accept this Medal with sincere thanks.

Transactions, American Geophysical Union Volume 31, Number 3 June 1950

THE WILLIAM BOWIE MEDAL

The William Bowie Medal was established by deed of gift in 1939 in honor of the distinguished and beloved geodesist and geophysicist whose name and likeness it bears. It is awarded annually in the name of the American Geophysical Union for distinguished attainment and outstanding contribution to the advancement of cooperative research in fundamental geophysics. The William Bowie Medalists, with the dates of their awards, are:

William Bowie (deceased)	1939
Arthur Louis Day	1940
John Adam Fleming	1941
Nicholas Hunter Heck	1942
Oscar Edward Meinzer (deceased)	1943
Henry Bryant Bigelow	1944
Jakob Aall Bonnevie Bjerknes	1945
Reginald Alsworth Daly	1946
Felix Andries Vening Meinesz	1947
James Bernard Macelwane	1948
Walter Davis Lambert	1949
Leason Heberling Adams	1950


THE WILLIAM
BOWIE MEDAL
UNSELFISH
COOPERATION
IN RESEARCH
TWELFTH AWARD TO
LEASON H. ADAMS
MAY 1, 1950
HYDROLOGY
OCEANOGRAPHY
SEISMOLOGY
VOLCANOLOGY
MAGNETISM
GEODESY

Twelfth William Bowie Medal (actual size is 2"×3 1/4")

Transactions, American Geophysical Union Volume 31, Number 3 June 1950

A RELATION BETWEEN MAXIMUM OBSERVED POINT AND AREAL RAINFALL VALUES

Robert D. Fletcher

Abstract--The envelope of world-record point-rainfall values for durations ranging from a few minutes to six months is shown to be closely approximated by the relationship that maximum depth is proportional to the square root of duration. The factor of proportionality is then shown to be a hyperbolic function of area, when the study is extended to include maximum observed depths of rainfall over sizes of area ranging from a point to over 200,000 sq mi.

When the millions of point-rainfall observations which have been recorded around the world are examined for the purpose of determining the relation of depth to duration of rain, the envelope of the plotted points emerges as a remarkably smooth and well-defined feature of the graph. In dealing with large ranges of values, it is often most convenient to make use of a log-log coordinate system. When such a system is employed in the present problem, logarithm of rainfall depth being plotted against logarithm of duration, it can be seen that the envelope very closely approaches a straight line. Figure 1 is a plotting of this sort in which, for convenience, an almost innumerable quantity of data was omitted in the lower rainfall brackets. The world-record and near record values have recently been tabulated by JENNINGS [1950]. Most of the values defining the envelope were recorded outside of the United States. The cluster of data within the duration range of six hours to three days, however, is concerned almost exclusively with storms which occurred within this country. So far as is known from an extensive survey and evaluation of available rainfall records, for each duration there is no higher value of rainfall which has been measured.

The envelope of the world-record values plotted in Figure 1 is of the form

$$R = kD^n \text{ . . . . } (1)$$

where R is rainfall depth, D is duration, and k is a proportionality factor. A strict envelopment of the plotted data produces, for n, a value of 0.49. Nearness of this exponent to 0.5 suggests use of the simple relation

$$R = k\sqrt{D} \text{ . . . . } (2)$$

Eq. (2) was fitted to the data, as indicated in Figure 1, the straight line envelope being drawn to pass through the value of 12 inches in 42 min which was observed at Holt, Missouri, in June 1947. When depth of rainfall is expressed in inches and duration in hours, the equation of the envelope becomes

$$R = 14.3\sqrt{D} \text{ . . . . } (3)$$

In order that the accuracy of (3) may be judged, percentages of the values determined by the envelope were computed for several of the observed record values and plotted directly below the respective points.

It was thought that certain locations in the world might exhibit the same trend of rainfall with duration which exists throughout the world as a whole. Selected for test were values observed along the Pacific Coast from northern California to the Canadian Border. It should be pointed out, however, that search for these data was not as exhaustive as that for world-record values, and that records for durations of less than one day were not explored. The highest values located, with durations ranging from one day to one year, are plotted also in Figure 1, and are enveloped by a straight line whose equation is

$$R = 2.9\sqrt{D} \text{ . . . . } (4)$$

and which provides a reasonably good fit to the data.

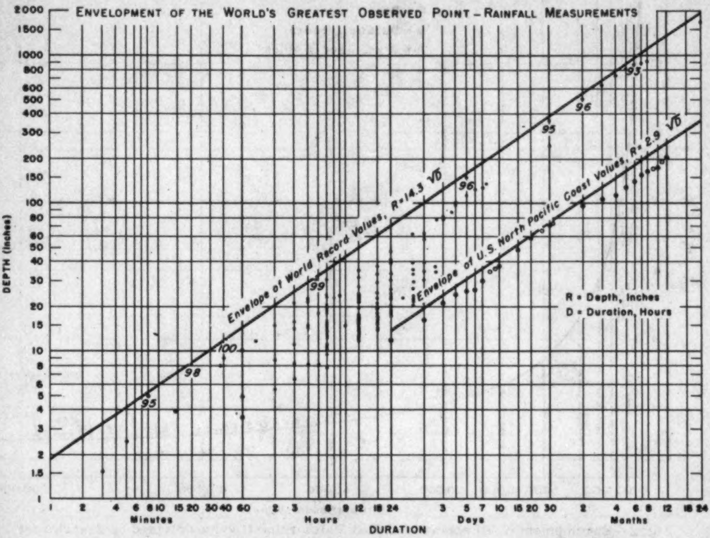


Fig. 1--Envelopment of the world's greatest observed point-rainfall measurements

While the Pacific Coast data fit the square-root relation quite well, it is likely that such is not the case in many other regions of the globe. In Missouri, for example, where 12 inches fell in 42 min, it seems improbable that seasonal amounts comparable with those which have been observed at Cherrapunji, India, (which provided the world-record values for durations at, and in excess of, five days) can ever occur.

Close fit of the maximum point-rainfall data to (3) raised the question as to whether maximum average depths of rain over area would bear the same type of relationship to duration. Preliminary to conducting tests for a possible relation, it was reasoned that square root of area (with dimensions of length to the first power), should appear in the equation relating rainfall depths with area and duration. Maximum depths of rainfall, averaged over sizes of area ranging from a point to over 200,000 sq mi, from storms in the United States which have produced record values were selected from the mass of storm-rainfall data compiled by the Corps of Engineers, Department of the Army, and collected and processed in cooperation with the U. S. Weather Bureau. The rainfall values, divided by square root of duration, were plotted against square root of area, the resulting array being shown in Figure 2. The data exhibited a marked tendency to possess a clearly defined envelope whose resemblance to a rectangular hyperbola is apparent. It also appears that the axes of the hyperbola are approximately parallel to and slightly displaced from the axes used in constructing the graph. Through trial determinations, a hyperbola of close fit was constructed. In developing the curve greatest weight was given the values for areas between a point and 20,000 sq mi, within which range most of the available values lie. The empirical equation form of the hyperbola which relates top values of depth of rainfall with area and duration thus becomes

$$R = \sqrt{D}\,[a + b/(\sqrt{A} + c)] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (5)$$

The quantity a represents the shift of the axis of rainfall divided by square root of duration, c represents the shift of the axis of square root of area, and b is the parameter representing the particular hyperbola chosen. Fitting (5) to the data produced the formula

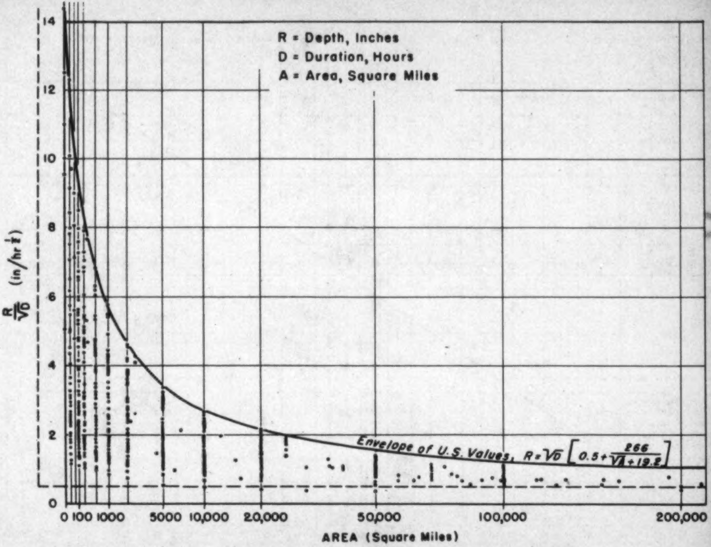


Fig. 2--Envelopment of all observed United States rainfall values divided by square root of duration (average depths over areas from a point to 212,000 sq mi)

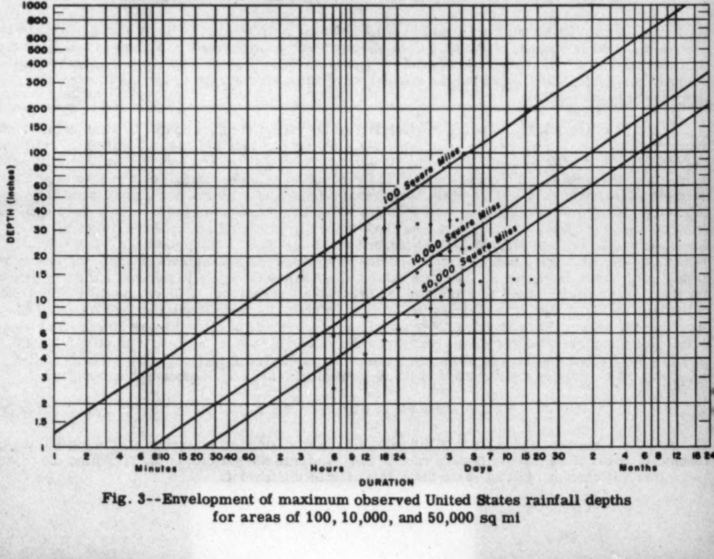


Fig. 3--Envelopment of maximum observed United States rainfall depths for areas of 100, 10,000, and 50,000 sq mi

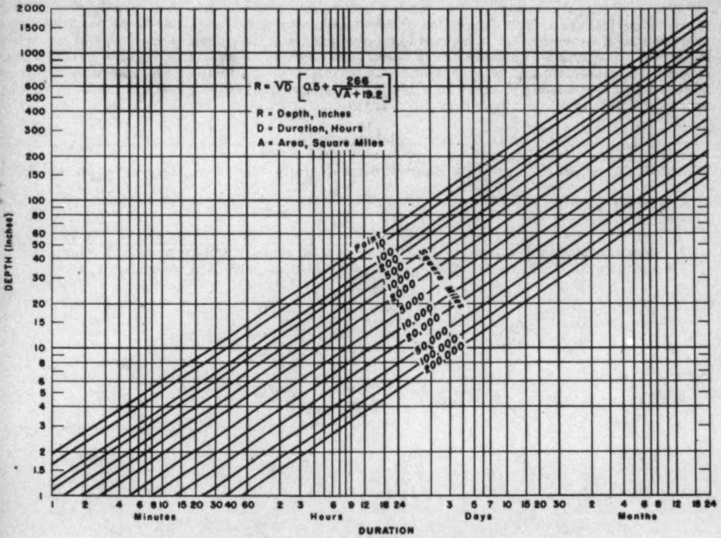


Fig. 4--Depth-area-duration curves enveloping observed rainfall depths over areas between a point and 200,000 sq mi

$$R = \sqrt{D}\,[0.5 + 266/(\sqrt{A} + 19.2)] \qquad (6)$$

in which depth is expressed in inches, duration in hours, and area in sq mi.

Eq. (6) produces a family of straight lines when reproduced on log-log coordinate paper. Three of these lines, for areas of 100, 10,000 and 50,000 sq mi, appear in Figure 3 together with maximum observed values of rainfall for the respective areas. For each of the lines the envelopment is close only for a restricted range of durations. It is believed that lack of close envelopment outside of these ranges is to a large degree caused by unequal samplings among the corroborating data. The basic data from which the curves were constructed were secured with very definite needs in mind - applications of the data to hydrologic design problems. Little need has been found for certain combinations, that is, depths for short durations over large areas and depths for long durations over small areas. Where the most useful combinations of duration and area are considered are found the closest fits to the empirical envelope. Equal samplings for all combinations would probably cause the sets of points to flatten out and more nearly approach the straight-line envelope. Whether data from such samplings would exceed the envelopes is a speculative matter.

In Figure 4 is given the set of enveloping depth-area-duration curves expressed by (6). The curves embrace durations between one minute and two years, and areas between a point and 200,000 sq mi. As has been demonstrated, the point-rainfall curve constitutes a very good envelope of observed values. Each curve for average depth over area closely envelopes values through only a small range of durations, a range, nevertheless, which is usually considered to be the most useful for practical purposes. Especially for long durations, the rainfall is not necessarily produced in a single storm in the normal sense of the word. Rather there may be a series of individual storms with considerable periods of fair weather between them. The equation is not concerned with how rainfall is distributed in time and space. It simply expresses the envelopment of maximum average depths of rainfall, over fixed areas, resulting from one or more individual observed storms.

The depth-area-duration relationship for maximum observed values has been developed through purely empirical reasoning. Derivation of (5) on a theoretical basis appears to be possible, however, and such a development is planned to be presented at a later date. Eq. (6) should not be used for estimating maximum possible precipitation until its physical tenability has been established. It appears a convenient means of expressing an envelopment of all values which have been recorded, however, and shows some promise as a tool for estimating the greatest average depths of rain which can fall in given durations over given areas.

Reference

JENNINGS, A. H., World's greatest observed point rainfalls, Mon. Wea. Rev., v. 78, pp. 4-5, 1950.

U. S. Weather Bureau,
Washington 25, D. C.

(Manuscript received January 23, 1950; presented at the Thirtieth Annual Meeting, Washington, D. C., April 20, 1949; open for formal discussion until November 1, 1950.)

Transactions, American Geophysical Union Volume 31, Number 3 June 1950

INVESTIGATIONS OF EVAPORATION FROM PANS

Gi-ichi Yamamoto

Abstract--The analogy between heat transfer and evaporation yields theoretical equations for evaporation from pans into still air and into moving air under turbulent and laminar boundary conditions. It is found that empirical equations obtained in a wind tunnel (using a water surface covered by blotting paper to prevent spraying) agree well with these theoretical results. The change between laminar and turbulent flow takes place suddenly and without any noticeable transition at a certain critical Reynolds Number. If the usual evaporation pan is used, however, as in Millar's experiments, the water surface becomes turbulent even for small Reynolds numbers and the empirical formula for evaporation is slightly different.

Introduction--It has been the main object of investigations of evaporation to find the relation between the rate of evaporation and wind velocity. Until some years ago, the rate of evaporation was taken to vary as the square root of wind velocity, but in recent investigations it was found that it is proportional to a power of wind velocity somewhat higher than 0.5 but not exceeding unity. For instance, HIMUS [1929] gives the following formula for the rate of evaporation E.

$$E = 0.02\,(e_0 - e_1)^{1.2} + 0.0446\,(e_0 - e_1)^{0.83}\,u_1^{0.77}\ \text{kg m}^{-2}\ \text{hour}^{-1} \ldots\ldots\ldots\ldots (1)$$

where e_0 and e_1 are respectively vapor pressures at the surface of water and in the overlying air current in mm Hg, and u_1 the wind velocity in m/sec. Also MILLAR [1937] made experiments in a wind tunnel using two different pans; the smaller one being 11.2 cm long and the larger one 50.8 cm long along the wind. According to him

$$E = 0.0326\,(c_0 - c_1)u_1^{0.725} \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

for the small pan and

$$E = 0.0190\,(c_0 - c_1)u_1^{0.77} \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

for the large pan, where E is the rate of evaporation in gm $\text{cm}^{-2}\ \text{sec}^{-1}$, c_0 and c_1 are respectively vapor concentration on the water surface and in the overlying air current in gm/cm^3, and u_1 is the wind velocity five cm above the water surface in cm/sec. (The system of units defined here will be used throughout in this paper.)

The experimental investigation of POWELL and GRIFFITHS [1935] is also worth noticing, because they studied not only the effect of wind velocity but also that of the size of the pan. According to them the rate of evaporation is proportional to the 0.85 power of wind velocity, and is almost independent of the width of the evaporating surface across wind except for very small wind velocities. However, it varies as a power of the length of the surface down wind, where the power ranges from -0.27 for small to -0.20 for large wind velocities.

Theoretical investigations on evaporation from a finite pan were carried out by JEFFREYS [1918], SUTTON [1934], and MILLAR [1937]. Millar's paper is interesting as he applied the momentum equation of von Kármán to the problem of evaporation.

In this paper are given brief summaries of the theoretical research carried out by the author [YAMAMOTO, 1941] and OGIWARA [1944], and the details of experiments which were carried out to verify the theory.

Theory--There exists a very close relation between heat transfer and evaporation. Making the usual assumption that dissipation of heat is negligible, the differential equations of steady state two-dimensional heat transfer in the case of laminar flow are

$$u\,\partial u/\partial x + v\,\partial u/\partial y = \nu\partial^2 u/\partial y^2 \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

$$\partial u/\partial x + \partial v/\partial y = 0 \quad \ldots\ldots (5)$$

$$u\,\partial t/\partial x + v\,\partial t/\partial y = (k/\rho c_p)\,\partial^2 t/\partial y^2 \quad \ldots\ldots (6)$$

where x and y are respectively coordinate axes measured along the surface and vertically; u and v, wind velocities in x and y directions, respectively; t, temperature of the air current; ν, kinematic viscosity; k, conductivity of air; ρ, density of air; and c_p, specific heat of air at constant pressure.

In the case of evaporation, (4) and (5) are the same but (6) is replaced by

$$u\,\partial c/\partial x + v\,\partial c/\partial y = d\,\partial^2 c/\partial y^2 \quad \ldots\ldots (7)$$

where c is the concentration of water vapor (absolute humidity) and d diffusivity of water vapor through air.

Hence if the boundary conditions also take similar forms in both cases, there exists perfect analogy between heat transfer and evaporation, and one can easily obtain the solution for c from the solution for t, if the latter is known. As is well known (4), (5) and (6) were solved exactly for the case of a flat plate by POHLHAUSEN [1921] under the boundary conditions; $t = t_o$, $u = v = 0$ at $y = 0$ and $t = t_1$, $u = u_1$, at $y = \infty$. According to him, the local rate of transfer of heat per unit area, q_x, is given by

$$q_x = -k(\partial t/\partial y)_{y=0} = (k/2)\alpha(\sigma)\,(u_1/\nu x)^{1/2}\,(t_o - t_1) \quad \ldots\ldots (8)$$

and the average rate for a plate of length x, q_{av}, is given by

$$q_{av} = (1/x)\int_0^x q_x\,dx = k\alpha(\sigma)\,(u_1/\nu x)^{1/2}\,(t_o - t_1) \quad \ldots\ldots (9)$$

where $\sigma = \mu c\rho/k$ (μ is coefficient of viscosity), and $\alpha(\sigma)$ is a function of σ. Pohlhausen computed $\alpha(\sigma)$ for values of σ between 0.6 and 15.0. Hence by analogy the local rate of evaporation per unit area, E_x, is given by

$$E_x = -d\,(\partial c/\partial y)_{y=0} = (d/2)\alpha(\sigma')\,(u_1/\nu x)^{1/2}\,(c_o - c_1) \quad \ldots\ldots (10)$$

and the average rate of evaporation E for a pan of length is given by

$$E = d \cdot \alpha(\sigma')\,(u_1/\nu x)^{1/2}\,(c_o - c_1) \quad \ldots\ldots (11)$$

where $\sigma' = \nu/d$, under the boundary conditions; $c = c_o$, $u = v = 0$ at $y = 0$ and $c = c_1$, $u = u_1$, at $y = \infty$. Introducing $d = 0.25$, $\alpha(\sigma') = .552$ (corresponding to $\sigma' = 0.6$), and $\nu = 0.15$ in (11), we have

$$E = 0.925\,u_1\,(u_1 x/\nu)^{-1/2}\,(c_o - c_1) \quad \ldots\ldots (12)$$

Now let us proceed to the case of a turbulent boundary layer. The differential equations for heat transfer are as follows

$$u\,\partial u/\partial x + v\,\partial u/\partial y = -(1/\rho)\partial\rho/\partial x + \partial(\ell^2|\partial u/\partial y|\,\partial u/\partial y)/\partial y + \nu\,\partial^2 u/\partial y^2 \quad \ldots\ldots (13)$$

$$\partial u/\partial x + \partial v/\partial y = 0 \quad \ldots\ldots (14)$$

$$u\,\partial t/\partial x + v\,\partial t/\partial y = \partial(\ell^2|\partial u/\partial y|\,\partial u/\partial y)\partial y + (k/\rho c_\rho)\,\partial^2 t/\partial y^2 \quad \ldots\ldots (15)$$

where ℓ is the mixing length according to Prandtl. For the case of evaporation we must replace (15) by

$$u\,\partial c/\partial x + v\,\partial c/\partial y = \partial(\ell^2|\partial u/\partial y|\,\partial u/\partial y)/\partial y + d\,\partial^2 c/\partial y^2 \quad \ldots\ldots (16)$$

Hence there also exists an analogy between heat transfer and evaporation in the case of a turbulent boundary layer. Further, if we neglect the effect of molecular viscosity, conduction, and diffusion, a quite permissible approximation, the analogy between t and c becomes perfect. LATZKO [1921] obtained an approximate solution of the above differential equations, leaving out molecular viscosity and conductivity, in the case of turbulent flow in a flat plate and obtained

$$q_x = 0.0285\,\rho c_\rho u_1 \,(u_1 x/\nu)^{-0.2}\,(t_0 - t_1) \ldots\ldots\ldots\ldots \quad (17)$$

which ELIAS [1929] finds in fair agreement with his experiments. Hence by analogy the following results are obtained for the local rate of evaporation E_x, and for the mean rate of evaporation E, for turbulent flow

$$E_x = 0.0285\, u_1 \,(u_1 x/\nu)^{-0.2}\,(c_0 - c_1) \ldots\ldots\ldots\ldots \quad (18)$$

and

$$E = 0.036\, u_1 \,(u_1 x/\nu)^{-0.2}\,(c_0 - c_1) \ldots\ldots\ldots\ldots \quad (19)$$

Experimental studies--The experimental results of Himus, Millar, and Powell and Griffiths seem to be not too different from these theoretical results, although there are some discrepancies. On the other hand in the cases of skin friction of a plate and heat transfer from a heated plate, the results derived from the boundary-layer theory have shown such good agreement with experiments, that it may be necessary to consider the cause of the discrepancies between theory and experiment in the case of evaporation. These discrepancies are probably due to the fact that the experiments were hitherto carried out using relatively complex boundary conditions and failed to measure certain necessary quantities such as surface temperature, whereas the theoretical treatment is at present limited to the idealized case in which, for instance, even the effect of the disturbance of the water surface by strong wind is not taken into account. Thus it will be necessary in testing the theory to satisfy the conditions demanded by it. In the present paper the results of such experiments are described and discussed.

Most previous experiments have been carried out with a narrow range of wind velocity so as to secure reliable information on the effect of this factor on the rate of evaporation. For instance, in Powell and Griffiths' experiments the maximum wind velocity was only three m/sec, and in Millar's experiment it was 8.9 m/sec. For this reason the author decided to measure the rate of evaporation using a wider range of wind velocity.

The wind tunnel used was of the Eiffel type, with a diameter of 25 cm at the end of the converging collector. A suction fan driven by a three horsepower induction motor was mounted at the end of the tunnel, and under ordinary circumstances could produce a constant wind velocity of 25 m/sec. But since it was essential to change the velocity, grids of various mesh sizes, some of which were covered by different kinds of cloth, were inserted at the mouth of the diffuser to obtain velocities ranging from about 0.5 to 25.0 m/sec. Although the wind velocity varied slightly each time, even if the same grid were inserted in the tunnel, this did not reduce the accuracy, because the actual velocity could be measured. Such a control method inevitably leads to an irregular spacial distribution of wind velocity at the working section of the tunnel. To avoid the possible inaccuracies which might result, small vessels were employed to measure the rate of evaporation.

Both a pitot tube connected to a manometer of Göttingen type and a hot wire anemometer were employed to measure wind velocity. The indication of the manometer, however, became inaccurate with small velocities, so that only the hot wire anemometer was employed in such cases. Platinum wire of 25 microns in diameter and two cm long was used.

The evaporation vessel used in the experiment is shown in Figure 1. It is mounted in the working section of the wind tunnel as shown diagramatically in Figure 2. The actual experimental surface consists of blotting paper as in the experiment of Powell and Griffiths. This is used instead of a true water surface since the latter becomes wavy with a strong wind and with increasing velocity begins to spray. These conditions are different from that assumed by the theory. Four vessels were used in the experiment, in which the wetted surfaces are respectively 5, 10, 15, and 20 cm along the wind, 10 cm across the wind and one cm in depth. These vessels were made of nickel coated brass, with the leading edge and the rear edge streamlined as shown in Figure 1. The experimental surface was covered by a wooden lid, the corners of which were attached with a tightly fitting rubber sheet to avoid unnecessary evaporation.

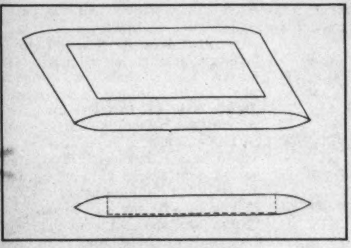


Fig. 1--Evaporation vessel

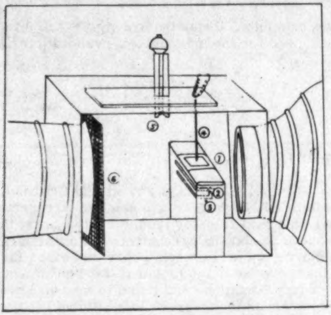


Fig. 2--Working section of wind tunnel; (1) evaporation vessel, (2) pitot tube; (3) hot wire anemometer, (4) thermo-junction, (5) Assmann psychrometer, (6) wind controlling grid

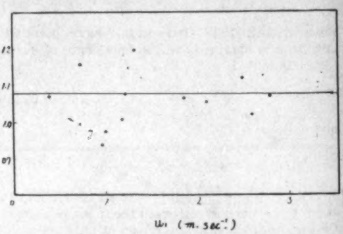


Fig. 3--Ratio of evaporation from surface of blotting paper E_b and from free water surface E_w as a function of wind speed u_1

This lid was taken off only during an experiment.

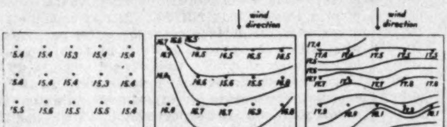
The rate of evaporation was obtained by measuring the weight of water lost during a certain time interval. A chemical balance of capacity, one kg, and sensitivity, one mg, was used. The time intervals were five-ten minutes depending on the type of experiment.

The surface temperature of the water is one of the important factors in evaporation measurements. It is difficult to measure the temperature of a free water surface, but with blotting paper it is not so difficult, as the movement of water is very much constrained. A thermo-junction made of copper wire 0.234 mm in diameter and constantan wire 0.254 mm in diameter were employed to measure the surface temperature.

The temperature and vapor concentration of the overlying air were measured by an Assmann psychrometer inserted in the working section as shown in Figure 2.

Comparison of evaporation from blotting paper and from a free water surface--This comparison was possible only with wind velocities less than 3.5 m/sec, because with larger velocity, water overflowed from the vessel. The results are shown in Figure 3, in which E_b and E_w are respectively the rates of evaporation from the surface of the blotting paper and from the free water surface. In this experiment the values of E_b are larger than those of E_w, and the ratio E_b/E_w seems to be independent of wind velocity within the range used. The mean value of $E_b/E_w = 1.08$. All data on the rate of evaporation in this paper are reduced to a true water surface by dividing the observed values by this ratio.

Distribution of surface temperature--It is interesting to measure the distribution of temperature on the experimental surface, because the rate of cooling may differ locally due to different rates of evaporation and the restrained motion of water in the vessel. This experiment was carried out with the ten cm long vessel. A few examples are shown in Figure 4. In Figure 4a it is shown that the distribution of surface temperature in still air is almost uniformly distributed over the whole surface as may be expected. Figure 4b shows that with the wind velocity 6.7 m/sec, the surface temperature is lowest at the leading edge and increases downwind. The temperature difference between the leading edge and the lee edge is 0.4°C. Figure 4c, with the wind velocity 14.8 m/sec, shows a temperature distribution is quite similar to the former case, except that the temperature difference between the leading



(a) (b) (c)

Fig. 4--Distribution of the surface temperature; (a) still air; (b) u_1 = 6.7 m/sec; (c) u_1 = 14.8 m/sec

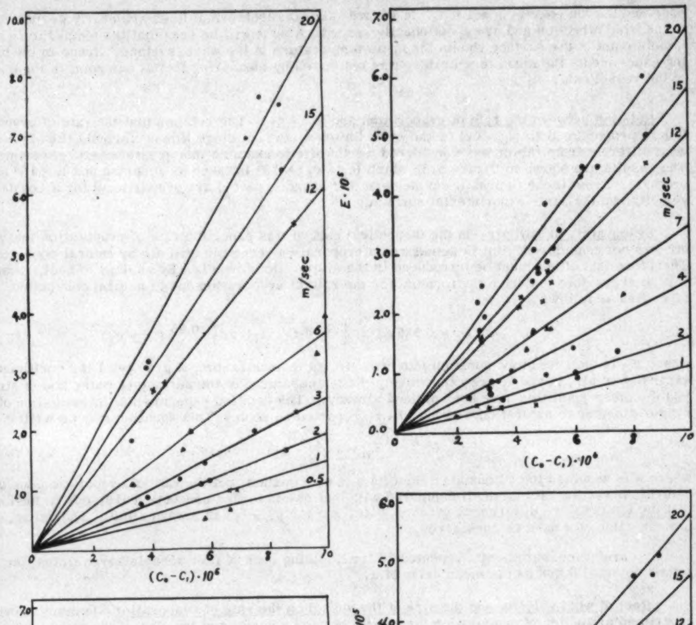


Fig. 5--Experiments with vessels of various lengths and wind velocities; each straight line shows the relation between E and $(c_0 - c_1)$ for the wind velocity indicated on the line (m/sec); (a) five-cm vessel; (b) ten-cm vessel; (c) 15-cm vessel; (d) 20-cm vessel

edge and the lee edge is now 1.0°C. In Figures 4bc, the isothermal lines are nearly perpendicular to the wind direction and are about equally spaced. Also it will be seen that the temperature of the middle point of the surface equals the mean temprrature of the whole surface. Hence in the following experiments the mean temperature was estimated by measuring that at one point in the middle of the vessel only.

Relation between the rate of evaporation and $(c_0 - c_1)$--The relation that the rate of evaporation is proportional to $(c_0 - c_1)$ is known as Dalton's law, but since Himus' formula (Eq. 1) is somewhat different from this it was considered worthwhile to examine this question again experimentally. The results are shown in Figure 5, in which $(c_0 - c_1) \times 10^6$ is taken as abscissa and $E \times 10^5$ as ordinate. From these figures it can be seen that E and $(c_0 - c_1)$ are proportional for a constant velocity and the same experimental surface.

Evaporation in still air--In the theoretical part of this paper the rate of evaporation into still air was not considered. But in actual cases evaporation occurs in still air by natural convection. Therefore this effect must be introduced in the evaporation formula. By analogy to heat transfer it is also possible to derive the formula for the rate of evaporation due to natural convection. It is written as follows

$$E_0 = 0.525\ (d/x) \left[x^3 g \delta (t_0 - t_1)/\nu d\right]^{0.25} (c_0 - c_1)$$

where E_0 is the rate of evaporation into still air, g the acceleration of gravity, δ the coefficient of expansion of air, t_0 and t_1 are respectively the temperatures of the surface of water and of air, and the other quantities have been defined already. (The detailed experimental investigation of evaporation due to natural convection will be reported on soon.) This equation may be written

$$E_0 = a\ (c_0 - c_1) \quad \text{.........................(20)}$$

where a is assumed to be constant. Strictly a is not constant, but the rate of evaporation due to natural convection (20) is small compared with that due to forced convection (19), due to the fact that the temperature differences between water and air $(t_0 - t_1)$ is always small. Therefore, only a mean value of a need be considered.

By careful measurements, repeated 28 times, using each of four vessels seven times, the author obtained 0.298 as the mean value of a.

Effect of wind velocity and the size of the vessel on the rate of evaporation--In many previous experimental studies of evaporation the effects of wind velocity and the size of the vessel were examined separately, and the experimental results combined in one empirical formula without investigating whether the boundary condition is laminar or turbulent. In the present investigation, for the purpose of testing (12) and (19), the effects of wind velocity and of the size of the vessel were examined at the same time in the form of the Reynolds Number, $(u_1 x/\nu)$. No attempt was made to derive one empirical formula; rather care was taken to determine whether each experiment belonged to a laminar or to a turbulent case. The following formula, an empirical modification of (12), (19), and (20), was used in examining the experimental data

$$E = \left[a + b u_1 (u_1 x/\nu)^n\right] (c_0 - c_1) \quad \text{...................(21)}$$

where a is the constant due to natural convection (0.298) and b and n are constants to be determined experimentally. Equation (21) is further transformed to

$$(1/u_1)[E/(c_0 - c_1) - a] = b\ (u_1 x/\nu)^n \quad \text{.................(22)}$$

Hence, putting $C_e \equiv (1/u_1)[E/(c_0 - c_1) - a]$, the relation between C_e and $u_1 x/\nu$ was examined. The experimental results are shown in Figure 6, where $\log_{10} (u_1 x/\nu)$ is taken as abscissa and $\log_{10} (1000\ C_e)$ as ordinate. Straight lines (1) and (2) in Figure 6 are given respectively by the equations

$$E = \left\{0.298 + 0.925\ u_1\ (u_1 x/\nu)^{-0.5}\right\} (c_0 - c_1) \quad \text{.............(23)}$$

$$E = \left\{0.298 + 0.036\ u_1\ (u_1 x/\nu)^{-0.2}\right\} (c_0 - c_1) \quad \text{.............(24)}$$

which are combinations of the theoretical results expressed by (12) and (19) and evaporation in still air from (20).

It will be seen that the results are in fairly good agreement with the line (1) for laminar flow in the range $\log_{10}(u_1x/\nu) < 4.75$. Beyond this value of Reynolds Number the results seem to follow the line (2) for turbulent flow. However, it may be necessary to use a larger critical value of Reynolds Number in order to define the turbulent case more exactly.

Experiments with larger vessels--Another experiment was carried out employing larger vessels, of which the experimental surfaces were respectively 40, 100, and 120 cm long in the direction of the wind, each 40 cm wide and five cm in depth, streamlined at both ends. The wind tunnel used was of the Göttingen type, which has an elliptic cross section of 1.5 × 1.0 m at the end of the converging collector. The range of wind velocity used was from 18 to 35 m/sec. The vessel was suspended in the working section of the wind tunnel in the same way as a model of a wing, and the rate of evaporation was measured by a balance mounted on the tunnel. The weighing was carried out in still air before and after the fan was on, and since the sensitivity of the balance was about one gram and the amount of water lost amounted to a few decagrams, it was possible to detect the loss of weight by evaporation. The results are shown in Figure 6 by the small squares. Combining these new results with those of former experiments it will be seen from Figure 6 that in the range $\log_{10}(u_1x/\nu) > 4.75$ the experimental results are almost in agreement with the line (2) for turbulent flow. But strictly it may be more correct to say that the results with larger vessels give slightly larger values than predicted by the line.

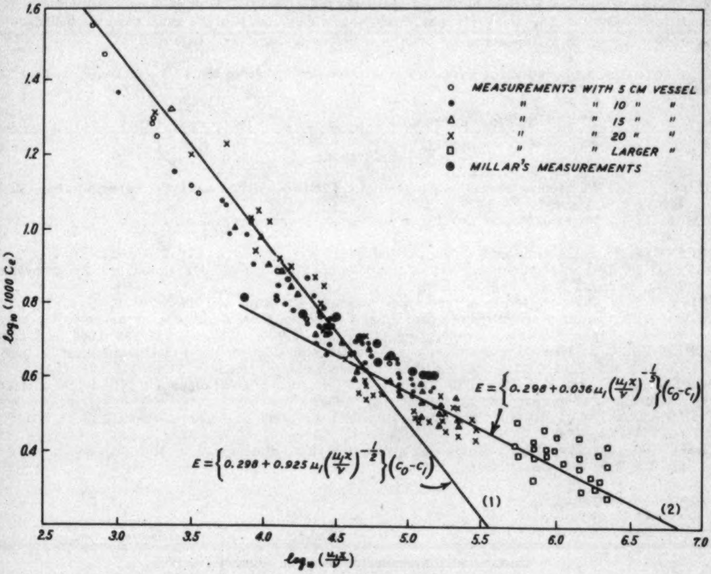


Fig. 6--Relation of Reynolds Number to rate of evaporation

It may be remarked that, while there is generally a very marked transition region noticeable when measuring the resistance of a plate, this was not found in the present experiment. This might be due to the fact that the leading edge of the experimental surface was not connected smoothly to the metal surface and some discontinuity existed between the two. Hence when the Reynolds Number became large, the entire surface suddenly changed from laminar to turbulent conditions.

Finally, Millar's experimental results were recalculated and entered in Figure 6. These results also seem to lie parallel to line (2) until $\log_{10} (u_1 x/\nu) \doteq 3.9$, but also seem to be slightly larger. In his experiments, a free water surface was used, so it might not be unreasonable to consider that some part of the discrepancy may be due to the disturbance of the free water surface by the wind which causes an increase of evaporation. One more point concerning Millar's experimental results is that in his experiment, the boundary layer seems to be turbulent even when Reynolds Number is below 10,000. This may be because the rim of the vessel containing water is naturally higher than the water surface, so that the air current over the surface of water may be turbulent with a small wind velocity. This suggests that measuring evaporation with the usual pans the boundary layer over the water will be turbulent even when Reynolds Number is below 10,000.

The rate of evaporation for a free water surface derived from Millar's experiment and the present theory is

$$E = [0.298 + 0.042\, u_1\, (u_1 x/\nu)^{-0.2}]\,(c_0 - c_1) \quad \text{. . . . . (25)}$$

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ON EARTH TIDES

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Abstract--This paper is divided into six main sections, the abstracts for which are as follows:

(1) This deals with the rigidity of the upper mantle of the Earth. A value is obtained for the effective modulus of rigidity of the Earth's upper mantle by using vector differences between tiltmetric observations at two suitably selected stations. By this method, the direct effects of Earth tides are almost entirely eliminated and the oceanic effects are accurately evaluated. (2) This deals with the particular motion of the Earth's surface near an active fault. Anomalous tilting diagrams obtained at six closely spaced stations near the sea are discussed in relation to the geologic structure of the surface ground. (3) Rigidity of the Earth as a whole is analyzed. A value of $1 + k - h$ is obtained from tiltmetric observations at a stable station far remote from the oceans, and values of the effective rigidity for the whole mantle and for the whole Earth are determined.

(4) This is a study of tidal variation of latitude. From data over long periods at six International Latitude Observatories, tidal latitude variations are analyzed and used to determine values of $1 + k - \ell$. Unexplainable effects peculiar to the observing stations and to the epoch adopted in analysis are noted. From data resulting from 300,000 pairs of star observations, the mean value of L was estimated as 1.20. (5) This analyzed the time variation of the elasticity of the Earth's crust. An unusually large variation with time in the Earth tidal amplitude observed at Makimine is discussed and is considered to be intimately related to the bending of the Earth's crust caused by the load of sea water in the nearby Hyûganada, an active center of large-scale sea earthquakes. (6) This is a study of the elastic anisotropy of the Earth's upper mantle. From an analysis of tiltmetric data obtained at several stations in Japan, it is suggested that the elasticity of the Earth's upper mantle in the N-S direction is 20 to 30 per cent smaller than that in the E-W direction. The study of longer series of observations and independent attacks on the problem are recommended.

CHAPTER I

Rigidity of the upper mantle of the Earth

Observations of Earth tide are generally made with tiltmeters, gravimeters, extensometers, and other instruments. Tiltmetric observations were most frequently used by the investigators in various parts of the world. From the observational data of Earth tide generated by the direct action of the Sun and Moon, it is theoretically possible to calculate the modulus of elasticity of the Earth under suitable assumptions. But, in the practical case, the observed tidal deformation of the Earth is the combined result of the direct effect of the Sun and Moon with the indirect effect of ocean tide. At a station near the sea especially the indirect effect is rather larger than the direct one. Concerning the indirect effect, the sea water, shifted by tide, acts as an increased load and an attracting mass, and moreover results in a change of the gravitational potential field caused by deformation of the Earth's crust. The tidal attraction of sea water for an observing station can be calculated from the actual tidal chart of the neighboring ocean, and both the bending of the Earth's crust by the tidal load of sea water and the distortion of the gravitational field can be related to the attraction component by Boussinesq's solution for deformation of an elastic plane, but they involve the elastic modulus of the crust as a parameter. Therefore, in analyzing observational data of Earth tide at one station near the sea, there are two unknown quantities to be determined: (1) the elastic modulus of the Earth as a whole, and (2) that of the Earth's crust, the thickness of which is related to the extent of the effective region of ocean tide. In tiltmetric observation of Earth tide at a continental station far distant from the oceans, we can determine the so-called "diminishing factor" D; that is, a ratio of observed value of tidal change to theoretical value for a hypothetical perfectly rigid Earth. From this D-value the modulus of elasticity of the whole Earth under a suitable assumption of density distribution in the interior of the Earth can be calculated. But the values of D hitherto obtained by many researchers showed large variations from 0.5 to 0.8, even if the extreme values are excluded. Likewise values for the rigidity of a homogeneous incompressible Earth vary from 3.6 to 14.6×10^{11} dynes/cm^2. On the other hand, the attraction component of sea water which is to be calculated from the actual tidal chart of the neighboring ocean also shows some uncertainty in the following points. In a practical calculation, we cannot neglect the effect of oceanic tides even at an observing station far inland. For example, in a previous observation in Japan, the sea region even 4000 km distant showed an appreciable effect. And unfortunately, as is well known, the values of tidal change in the open sea are very uncertain. Thus in an analysis of the observational data of Earth tide at one station, especially at one near the sea, we can not avoid these uncertainties in value of D and attraction component of sea water, as they are always observed together and therefore are disturbing one another in the course of their analytical determination.

In order to avoid these undesirable mutual disturbances, we worked on the differences of the observed data of two selected stations. In this way the direct effect of the Sun and Moon was eliminated, and, moreover, the effect of distant sea water was reduced to a negligible quantity. For practical purposes, we selected Kamigamo Geophysical Observatory and Aso Volcanological Laboratory.

Kamigamo (λ = 135° 42′E, ϕ = 35° 02′N) is situated in the northern part of the city of Kyoto. Its observing room is nine meters below the ground surface. Aso (λ = 131° 00′E, ϕ = 32° 53′N) is in the district of Volcano Aso in Kyûshû Province. Its observing room is 22 m deep. Observations of Earth tide were made with "fused silica" tiltmeters of horizontal pendulum type of Zöller suspension. The periods of observation used were January 1939-January 1940 for Aso and May-December 1939 for Kamigamo.

From the observed data at these two stations, we extracted tidal components of lunar semidiurnal M_2 and lunar diurnal 0_1 by the method of harmonic analysis and then calculated respectively the vector differences for the two stations. Of the largest component tide M_2 we have a diagram of tilting at the two stations as seen in Figure 1. Next, the approximate values of direct effect upon the two stations were calculated and then their difference was taken. In this way we could see that the direct effect from the vector differences was almost cancelled, thus the uncertainty of D-value had little effect upon the determination of indirect oceanic action. In this way, we could separate the two unknowns, the direct and indirect effects, and discuss the pure indirect effect in a form of difference values alone. On the other hand, we calculated, for the two stations, the attraction components of ocean tide using the actual tidal chart of the Pacific Ocean extending for 4000 km and then similarly took their differences, taking into consideration the relation of distance between the sea region and the station. At once it was observed that the main part of oceanic tidal effect is caused by tidal water within 1000 km. Therefore, uncertainty in values of ocean tide in remote regions of the open sea would be negligible in the differences of the attraction components.

We can treat the problem as one of pure oceanic action by the above process. Boussinesq's solution for deformation of an elastic plane by a surface load shows that there exists a simple relation containing the plane's modulus of elesticity as a parameter between bending of the elastic plane and attraction of load. But in the practical case of the Earth, the elastic modulus is not uniform everywhere in the Earth, and we usually adopt a general assumption that the physical constant in the Earth's interior is expressed as a function of depth only. Hence we also assume in our case that the two unknown effects of ocean tide, bending component, and deformation of potential field, have a simple relation to the calculable attraction component, but containing the elastic modulus and the attraction distance as two parameters. In Figure 2 we can see that the attraction component and the observed oceanic effect are related in a simple way; hence we tried by a method of

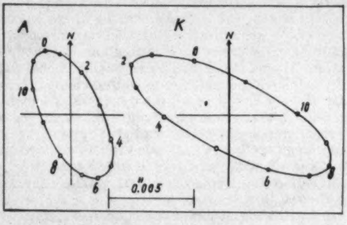

Fig. 1--Observed values of M_2-term at Aso (A) and Kamigamo (K)

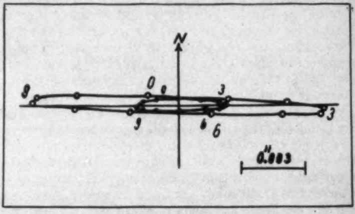

Fig. 2--Difference values of M_2-term; observed oceanic effect (outer curve) and calculated attraction of tidal water (inner curve) successive trial and error to determine the most suitable law to relate the two, taking, of course, distance of attraction into consideration. Finally, we obtained a most suitable formula connecting the attraction component and the observed oceanic term under an assumption that the elastic modulus constantly increases up to a certain depth of the Earth. Practically speaking, Boussinesq's solution for a homogeneous non-gravitating elastic plane gives the following relation between attraction of load A and amount of bending of surface of an elastic plane B

$$B = mA \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

$$m = (\lambda + 2\mu)g^2/4\pi\mu(\lambda + \mu)G \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

where λ and μ denote Lamé's elastic constants, g the surface gravity acceleration of the Earth, and G the universal gravitation constant. In (1) and (2) m is not a constant but is a function of the distance of action, because the participating depth of the Earth's crust in its deformation depends on the action distance, and the elastic modulus of the Earth is a function of depth.

Next, concerning the effect C of change of gravitational field caused by surface load, it is considered to be small compared with the other two effects from a rough theoretical estimation and would be expressed approximately by a relation $C = -n \cdot A$ (n = a positive constant), because the C-effect is considered to be an existence of negative masses, so to speak, contrary to the effect of the attracting masses (A-effect). Therefore, the total oceanic effect connected with the A-effect would be expressed in a form of $(1 + m - n)A$. The next problem is to find the most suitable formula to fit the relation to the observed oceanic effect $(1 + m - n)A$, taking into consideration the distance of action. Thus, we finally obtained as most suitable values of m and n the following numerical expressions

$$m(r) = 12.6/(r + 3.0) \;, \qquad n = 0.5 \quad \ldots\ldots\ldots\ldots\ldots \quad (3)$$

where r is the arc distance expressed in angular degree between the station and the sea region involved. The elastic modulus for every successive depth of the Earth to about 1000 km could be determined as corresponding to the effective distance of every sea region by these procedures. Next, by summation of effects of sea regions, the ratio of the total effect of bending component ΣmA to the total attraction effect ΣA would give a mean, or, more appropriately expressed, an effective modulus of elasticity of the Earth within a certain depth, about 1000 km in our case. This is because the remote open ocean beyond 1000 km has no appreciable influence upon the station in our difference method, and the depth of the layers of the Earth participating in deformation by tidal load of ocean is estimated roughly to be within the order of 1000 km. The above reasoning will be supported by estimating the effective depth of action of an elastic plane for a surface load in Boussinesq's solution and considering the linearity of the increase of elastic modulus with depth in the case of the Earth.

In this way we obtained $m = 2.71$ as the effective value of m for a layer of the Earth to some 1000 km depth, and then determined the effective rigidity of that layer by (2). Taking $g = 979.6 \text{cm/sec}^2$, $G = 6.670 \times 10^{-8}$ dynes cm^2/gr^2, Poisson's ratio $\sigma = 0.27$, and so $(\lambda + 2\mu)/(\lambda + \mu) = 2(1 - \sigma) = 1.46$, we have $\mu_e = 6.17 \times 10^{11}$ dynes/cm^2.

On the basis of seismological evidence, we can divide the Earth generally into the following parts: crust (surface-60km), upper mantle (60-1200 km), lower mantle (1200-2900km), and central core (2900-center).

In our present problem we are dealing with the layer corresponding to the upper mantle of the Earth in a seismological sense, so we can say that the effective modulus of rigidity of the upper mantle of the Earth is estimated to be 6.17×10^{11} dynes/cm^2. This value is nearly equal to that of platinum under a normal condition of pressure and temperature on the Earth's surface.

CHAPTER II

Particular motion of the Earth surface near an active fault

By an analysis of observational data of Earth tide with tiltmeters, we can determine the elastic modulus of the Earth and the Earth's crust as shown in Chapter I. At an observing station, especially one close to the sea, the amount of deformation of the Earth's surface caused by the tidal load of neighboring sea water is enormously large compared with the other effects. Therefore, we can hope to detect the microstructure of the superficial layer of the Earth by tiltmetric observations at a station near a sea. We set up tiltmeters for this purpose at six points in Beppu ($\lambda = 131°\ 30'$E,

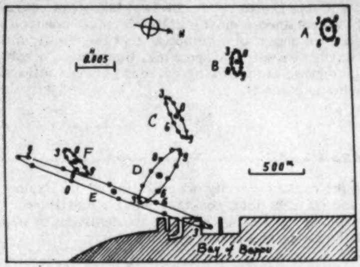

Fig. 3--M_2-tide observed at Beppu

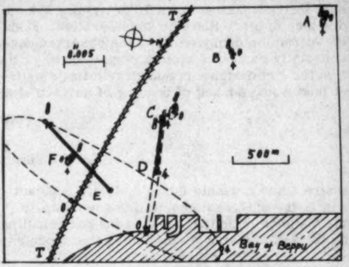

Fig. 4--Earth tide anomaly at Beppu

ϕ = 33° 16'N), the city of hot springs in Kyushu Province. The six points are several hundred meters apart and the points nearest and most remote from the sea are 400m and 1800 m respectively, from the shore of the Bay of Beppu as shown in Figure 3.

The observations of two years duration (1937-1939) were made simultaneously by six sets of fused silica tiltmeters, with the observing rooms of each point being several meters below the surface of the ground. Analyses of the data obtained from each of the six points showed the very strange result in that the tilting diagrams of the stations bear no resemblance to each other (see Fig. 3), contrary to our expectancy that the tilting motion caused by tidal load of sea water, if observed at such adjacent points, would normally show similarity. Then we calculated for each point the direct effect of the Moon and the indirect effect of neighboring ocean and especially the tidal sea water of Beppu Bay in the usual way, and found that these calculated effects were small compared with the observed values, especially for two points, E and D. We subtracted all commonly known effects of the Moon from the observed data and finally, as residuals or, more appropriately expressed, anomalies of Earth tide, obtained the tilting diagrams as seen in Figure 4. We denoted the tilting amount and direction by thick arrows in Figure 4 at the lunar hour when Beppu Bay is in high tide, the eighth lunar hour referred to the central meridian (135°E) of Japan. We concluded that the anomalous tilting motions apparent in Figure 4, are peculiar to the observing points themselves.

As an interpretation of the anomalous behavior of the Earth's surface at Beppu, attention is given to the geological structure of that region. Referring to Figure 4, there is a great active fault as indicated by a double line (T-T) running in SE-NW direction through the southern part of the city of Beppu, which distinctly divides the area geologically. The southern part is a region of mountains of ancient volcanic rock and the northern is an alluvial plain. A subsiding movement of large scale of the alluvial plain relative to the southern mountain region is geologically detected, showing a sharp difference in elevation of about fifty meters. Movement is continuing to the present time, but on a small scale. An actual example of that movement is described in history. On September 4, 1596, the small island of Uryû in the Beppu Bay sank into the sea bottom in a moment by a violent subsiding movement of the northern part of that fault. And, moreover, the activity of this fault has been revealed by recent precise leveling, which proves that the upheaval of the southern part or the subsidence of the northern part of this fault, or both, continue to the present time, with the fault as a boundary.

Comparing the geological structure of Beppu with the observed anomalies of Earth tide, the peculiar motions of the ground generated by tidal loading of sea water of the Beppu Bay at some stations can be explained to a certain extent, especially at E and D. It seems to us that when the alluvial plain is bent by the loading of tidal sea water the fault line behaves like a free boundary of an elastic plate, so that some points near both the fault and the sea shore tilt to the fault line rather than to the direction of the Bay. The point F on firm ground of volcanic rock is affected very little by this fault. Thus the mosaic structure of the surface layer of the Earth can be investigated by a dense net of tiltmetric observations at points near the sea.

CHAPTER III

Rigidity of the Earth

According to Love's theory, denoting tide-generating potential, gravity acceleration, radial displacement, and additive potential caused by deformation, all on the Earth's surface, by W_2, g, U_0, and V_d, respectively, we can define the so-called Love-numbers h and k as $U_0 = hW_2/g$, $V_d = kW_2/g$. Therefore, for the real elastic Earth, the tide-generating potential W_a is not equal to the original value of W_2, but

$$W_a = (1 + k - h)W_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

Taking a ratio of the actual potential W_a to the theoretical W_2, we have

$$W_a/W_2 = D = 1 + k - h \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (5)$$

That is, the so-called diminishing factor is a ratio of W_a to W_2, or in other words, a ratio of the tidal amplitude of the actual Earth to the theoretical. And with regard to h and k, we can obtain from a theoretical consideration a simple relation of the elastic modulus of the Earth with h and k when the Earth is assumed to be homogeneous and incompressible

$$k = 3h/5 \ , \qquad h = 5g\rho a/(2g\rho a + 19\mu)$$

where a, ρ, and μ represent the mean radius, the uniform density, and rigidity of the Earth respectively. In such a simple case, we can at once determine the uniform rigidity of the Earth by the D-value obtained in tiltmetric or other observation of Earth tide.

The above consideration is concerned with the pure direct effects of the Sun and Moon. But in the practical case, as was seen in Chapters I and II, the observed values always contain the indirect effects of ocean tide. As mentioned, separation of the two effects is very difficult, and, especially for an observing station near the sea, it is impossible to extract accurately the direct effect only and discuss the D-value, because the indirect effect is much larger than the direct at such a station. Now tabulating the D-values obtained from tiltmetric observations by previous investigators in various places of the world, we can see how deeply the D-values everywhere suffer from the undesirable effect of the indirect action. Since the first observation at Strassburg in 1892 by von REBEUR-PASCHWITZ [1897], there have been about twenty investigations of Earth tide by tiltmeters, giving D-values ranging from 0.26 to 1.10. Among these, only MICHELSON'S [1919] value (0.690) is considered to be little affected by the indirect effect of ocean tide, because his observation was made under a very favorable condition with the tiltmeter of water-tube type at Yerkes Observatory, which is more than 1100 km distant from significant tidal-water effects. The other observations are thought to be more or less affected by the neighboring sea, as the observing stations were not far distant from the ocean.

In order to obtain an accurate value of D on the Asiatic Continent, we selected an observing station named Barim (λ = 122° 10′E, ϕ = 48° 18′N) in the northwestern part of Manchuria. Barim is more than 1000 km distant from the surrounding seas and 2000 km from the Pacific Ocean, the influence of which, if any, is great compared with that of the nearer seas. The observing station is on top of a hornfels mountain 790 m high, and the observing room is situated at the end of an adit 70 m long and 44 m below the ground surface. The adit was formerly prospected as a copper mine. The observation was made during June - September, 1941, with two components of fused silica tiltmeters, the sensitivities of which were kept constant during the time of observation at about 0.01″/mm for an optical distance of two meters in photographic recording. The condition of the observing room was very stable and not at all disturbed by thermal and other undesirable effects; thus we could obtain a very genuine record of Earth tide proper.

Analyzing harmonically the observed data and applying all corrections accompanying the method of analysis and the orbital element of the Earth, but not correcting for the inclination of the Moon's orbit to the equator (J = 18°.36), we found values of component tides in each component time referred to the Barim meridian (λ = 122° 10′E) set forth in Table 1. Next, the indirect effects of the surrounding oceans were calculated by the method described in Chapter I, adopting the values of elasticity of the Earth's mantle already obtained. The total effects of ocean tide for J = 18.36° have the numerical values shown in Table 2.

Thus, it can be said that the oceanic effects are very small compared with the observed values and of the order of magnitude of observational errors. Applying these insignificant corrections of

Table 1--Values of component tides at Barim

Term	Component	Magnitude
M_2	NS	0.00565cos (2t-349.5)± 0.00009
	EW	0.00720cos (2t- 91.7)± 0.00013
S_2	NS	0.00243cos (2t-349.2)± 0.00004
	EW	0.00282cos (2t- 85.9)± 0.00008
O_1	NS	0.00021cos (t-322.2)± 0.00008
	EW	0.00228cos (t- 96.2)± 0.00006

Table 2--Effects of ocean tide at Barim

Term	Component	Magnitude
M_2	NS	0.00016cos (2t-175.0)
	EW	0.00018cos (2t-228.6)
S_2	NS	0.00007cos (2t-173.0)
	EW	0.00009cos (2t-287.0)
O_1	NS	0.00007cos (t-209.6)
	EW	0.00021cos (t-324.9)

oceanic tide to the observed values, the final values of pure direct effect of the Sun and Moon, together with the calculated values of a theoretical perfectly rigid Earth referred to the latitude and longitude of Barim and inclination $J(= 18.36°)$ at the time of observation are shown in Table 3. The tilting diagrams are given in Figure 5.

The N-S component of O_1 is theoretically equal to zero at a station of 45° latitude, and in reality observations at Barim ($\phi = 48° 18'$) gave a very small value comparable with observational errors; therefore it is appropriate to exclude it in our discussion.

Taking for each component tide, the ratio D of the amplitude of the observed to the theoretical and their phase difference κ in angular degree, we have the numerical values shown in Table 4.

It can be concluded that in the phenomena of Earth tide the amplitude ratio of the observed to the theoretical on the Asiatic Continent shows a numerical value of 0.661 and that there is no phase difference between the body in action and the generated tide.

Since the first treatment of a homogeneous incompressible Earth by Kelvin in 1863, many theoretical investigations were made by Love, Darwin, Herglotz, Prescott, Schweydar, Hoskins, Prey, Jeffreys, and others. They tried to determine the distribution of elastic modulus in the interior of the Earth, using the observed data of Earth tide, seismic wave, variation of latitude, and other allied phenomena, under various assumptions as to the internal structure of the Earth. In this paper we adopted Herglotz's method modified by JEFFREYS [1926] for a two-layer Earth which is compatible in a first approximation with the Earth structure deduced from observations of seismic waves. To the two layers which correspond to the mantle and the Central Core defined in Chapter I, we assigned the values shown in Table 5 according to Jeffreys, considering the mean density (5.53), the moment of inertia of the Earth, and the depth of the boundary of the two layers (2900 km). Then we calculated the rigidity of the two layers from the observed value of D, using the Herglotz-Jeffreys' equation that relates h and k with μ and δ (ratio of the rigidity of the central core to that of the Mantle), by assuming a definite value for δ. From the seismological evidence, $\delta = 0$ is first used. Then we compute the value of μ as the rigidity of the Mantle including its upper and lower parts.

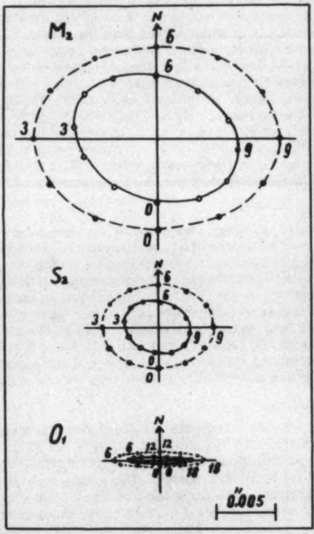

Fig. 5--Observed values of Earth tides at Barim (inner curves) and calculated theoretical values (outer curves)

Table 3--Direct tidal effect of the Sun and Moon, and calculated values at Barim

Term	Component	Direct effect	Theoretical effect
M_2	NS	0.00582cos (2t-349.6)	0.00814cos 2t
	EW	0.00733cos (2t- 90.8)	0.01091sin 2t
S_2	NS	0.00250cos (2t-349.8)	0.00365cos 2t
	EW	0.00291cos (2t- 86.6)	0.00489sin 2t
O_1	NS	0.00025cos (t-336.6)	0.00061cos t
	EW	0.00243cos (t- 99.9)	0.00395sin t

Table 4--Values of D and κ

Term	Component	D				κ			
		Observed	Average	Weight	Weighted mean	Observed	Average	Weight	Weighted mean
M_2	NS	0.715				-10°.4			
	EW	0.672	0.694	2		+ 0.8	-4.8	2	
S_2	NS	0.685				-10.2			
	EW	0.595	0.640	1		- 3.4	-6.8	1	
O_1	NS	(0.411)[a]				(-23.4)[a]			
	EW	0.615	0.615	1	0.661 ±0.024	+9.9	+9.9	1	-1°.6 ±3°.8

[a] Rejected values.

Table 5--Values (after Jeffreys) of density and rigidity

Layer	Density	Rigidity
	gr/cm^3	dynes/cm^2
Mantle	4.27	μ
Core	12.04	$\delta\mu$

Table 6--Effective rigidity

Portion	μ
	10^{11} dynes/cm^2
Upper mantle	6.17
Whole mantle	13.95
Whole Earth	6.56

Adopting 0.661 as the D-value, we obtained 13.95×10^{11} dynes/cm^2 as the modulus of rigidity of the mantle. This means that if we adopt the two-layer structure as a model of the Earth and assume that the rigidity of the central core is zero, then the mantle shall show in a sense, a mean, or, more appropriately expressed, an effective rigidity as obtained above against an external deforming force, fitting to the observed tidal deformation on the actual Earth's surface. In such a sense, we may properly call this rigidity "the effective rigidity of the mantle." Next, putting $\delta = 1$ in the Herglotz-Jeffreys' relation, we obtained the value of 6.56×10^{11} dynes/cm^2 as the effective rigidity of the whole Earth in the sense above mentioned. Thus adding the result obtained in Chapter I for the rigidity of the upper mantle, we conclude that the effective rigidity of the upper mantle (60-1200 km deep), the whole mantle (60-2900 km deep), and the whole Earth against external deforming forces are as shown in Table 6.

CHAPTER IV

Tidal variation of latitude

It is well known that the latitude at any position on the Earth's surface, precisely determined astronomically by Talcott's method, shows a periodic change of Eulerian nutation, containing two main terms of annual and 14-month periods. Observational data of latitude thus determined also contain terms of tidal effects by the Sun and Moon, although the amplitudes of these terms are very minute (magnitude of 0.″01) in comparison with original latitude variation (magnitude of 0.″3). But we can extract such terms of tidal variation from the observational data of latitude by considering a sufficiently long period. A five-year period, for example, contains nearly ten thousand pairs of star observations with a total mean error of about 0.″001. As previously mentioned, the tide-generating potential on the elastic Earth is expressed in a form of $(1 + k)W_2$, in which W_2 is the original tide-generating potential. Now putting s as the meridional displacement of an observing station caused by Earth tide, we can define a symbol ℓ as in

$$s = (\ell/g)\,\partial W_2/\partial\phi$$

where ϕ denotes the latitude. Hence change of latitude caused by s-displacement may be expressed as

$$-(\ell/ag)(\partial W_2/\partial\phi)$$

where a is the mean radius of the Earth.

Together with the additive potential kW_2, the total change of latitude on the elastic Earth caused by Earth tide can be expressed as

$$[(1 + k - \ell)/ag](\partial W_2/\partial\phi)$$

Table 7--Results of analyses of residual latitude variations

Station	a	κ	Epoch analyzed	Investigator
	″	°		
Carloforte, Italy	0.0088	-4	1900-1908	Shida-Matsuyama
	0.0054	-2	1900-1912	Przybyllok
	0.0065	-3	1900-1912	Przybyllok
	0.0118	-26	1912-1923	Nishimura
	0.0082	-3	1923-1931	Kawasaki
Mizusawa, Japan	0.0077	+28	1900-1912	Przybyllok
	0.0032	-100	1912-1923	Nishimura
	0.0114	-11	1923-1931	Kawasaki
Ukiah, California, U. S. A.	0.0024	+8	1900-1912	Przybyllok
	0.0105	+11	1912-1923	Nishimura
	0.0123	-21	1923-1931	Kawasaki
Gaithersburg, Maryland, U. S. A.	0.0074	+21	1900-1912	Przybyllok

Taking the ratio of this to the corresponding expression for a hypothetical perfectly rigid Earth, we have

$$L = 1 + k - \ell \quad \ldots\ldots\ldots\ldots \quad (6)$$

The ratio L has an intimate relation to D (D = 1 + k - h) in tiltmetric observation. In the simple case of a homogeneous incompressible Earth, ℓ and h, which are respectively related to the meridion and radial displacements of the Earth surface caused by Earth tide satisfy the relation

$$\ell = 3h/10 \quad \ldots\ldots\ldots\ldots \quad (7)$$

To extract the tidal effect from the observational data of latitude, the original large change of latitude of annual and 14-month periods should be eliminated by subtracting a general smoothed value of their change from the individual values of observed stars, and the minute residuals thus remaining may be harmonically analyzed into a lunar semi-diurnal term and into other terms.

Since the first calculation by SHIDA and MATSUYAMA [1912] several researchers, such as PRZYBYLLOK [1921, 1923], STETSON [1929, 1933], KAWASAKI [1935], NISHIMURA [1937], JONES [1939], and others, made their calculations on the same item, but used slightly different analyzing methods and different periods of observation for various stations, all of them covering about ten years' observations. Table 7 presents, for their results, the values of a and κ in the expression $a \cos (2t - \kappa)$ referred to the M_2-term.

We can see in this table how far the individual values obtained by various researchers differ, even for the same station. The theoretical value for a station at 39° 08′ latitude calculated for a perfectly rigid Earth should be $0.''0077 \cos 2t$ and for the real Earth should be $0.''0077 (1 + k - \ell) \cos 2t$, t being the hour angle of the lunar term. Therefore, if there were no error in calculation and no disturbances of oceanic, local, and other unknown character, all of the obtained values should be coincident within the errors of observation and calculation. But this is not so; hence, we must pursue our research regarding the sources of these discrepancies.

Along these lines, we tried to detect the tidal variation of latitude from all observed data now available of the International Latitude Service, applying to them the same analyzing method. As a first step, we divided the whole epoch into four divisions as shown in Table 8.

As a final result for each epoch and station, Table 9 gives the values we obtained for the cosine form of the M_2-term, referring the M_2 hour angle to the meridian of each station.

Comparing these results with those of previous researchers, we noticed that the results obtained by various methods from the same data are nearly the same within their mean errors. Therefore, it can be said that wide discrepancies as seen in the results obtained for different epochs and different stations of the same latitude, may be attributed not to errors of calculation, but to the station itself and the epoch adopted. We shall discuss this point next.

Table 8--Divisions of the whole epoch

Station	Longitude	Divisions			
		I	II	III	IV
	° ′	years	years	years	years
Mizusawa, Japan	141 08 E	1900.0-06.0	06.0-12.0	12.0-22.7	22.7-31.0
Carloforte, Italy	8 19 E	1900.0-06.0	06.0-12.0	12.0-22.7	22.7-31.0
Ukiah, California	123 13 W	1900.0-06.0	06.0-12.0	12.0-22.7	22.7-31.0
Tschardjui, U.S.S.R.	66 53 E	1900.0-06.0	06.0-12.0	12.0-19.4	......
Cincinnati, Ohio, U.S.A.	84 25 W	1900.0-06.0	06.0-12.0	12.0-16.0	......
Gaithersburg, Maryland, U.S.A.	77 12 W	1900.0-06.0	06.0-12.0	12.0-15.0	......

Table 9--Values of M_2-term

Station	Epoch division	M_2		
		″	°	
Mizusawa	I	0.0011 cos (2t -	75)	±0.0025
	II	0.0067	329	15
	III	0.0002	275	14
	IV	0.0079	2	20
Tschardjui	I	0.0131 cos (2t -	10)	±0.0030
	II	0.0076	26	20
	III	0.0088	0	41
Carloforte	I	0.0083 cos (2t -	357)	±0.0010
	II	0.0061	352	12
	III	0.0105	329	7
	IV	0.0116	0	13
Gaithersburg	I	0.0072 cos (2t -	77)	±0.0017
	II	0.0035	286	24
	III	0.0061	319	23
Cincinnati	I	0.0183 cos (2t -	26)	±0.0013
	II	0.0136	9	24
	III	0.0185	282	30
Ukiah	I	0.0049 cos (2t -	47)	±0.0023
	II	0.0056	332	26
	III	0.0098	357	22
	IV	0.0111	1	17

For the station effect, we may mention the indirect effect of ocean tide, the meteorological effect peculiar to the station, the difference, if any, between the continental and coastal situation, and other unknown disturbances. For the epoch effect, it is possible that there may exist unknown secular variation of some astronomical or geophysical elements concerned in latitude determination, or that the epochs adopted (about ten years) were too short to determine the accurate value of minute tidal effect from observed data of original latitude variation, which are considered to be greatly disturbed by many unknown effects. As a first step, may be discussed the mean value of the tidal variation of latitude obtained from the data in the whole epoch of observations now available.

The Report of the International Latitude Service, volume 8 covering 1922.7-1935.0, by H. Kimura issued from the Mizusawa Observatory, enables us to analyze the full data of a long epoch extending from 1900 to 1935, including the individually observed values of about 300,000 star pairs. Thus, together with the results obtained previously, we secured the results shown in Table 10 and Figure 6 as the total mean values of the whole epoch at each of the six International Latitude Observatories.

Next the indirect effects of ocean tide for latitude variation were calculated containing terms

Table 10--Results of analysis of tidal variation of latitude

Station	M_2	Epoch analyzed
	″ ° ″	
Mizusawa	0.0029 cos (2t-352) ± 0.0010	1900-1935
Tschardjui	0.0102 cos (2t-353) ± 0.0015	1900-1919
Carloforte	0.0082 cos (2t-347) ± 0.0005	1900-1935
Gaithersburg	0.0019 cos (2t- 17) ± 0.0016	1900-1915
Cincinnati	0.0125 cos (2t-354) ± 0.0014	1900-1916
Ukiah	0.0077 cos (2t- 0) ± 0.0005	1900-1935

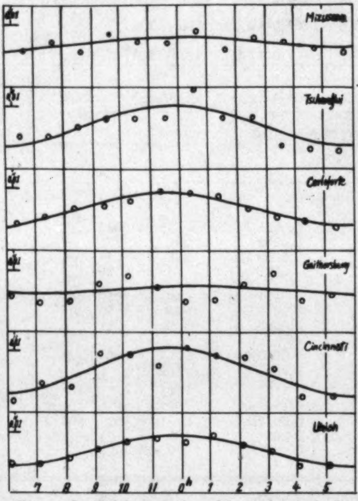

Figure 6--Tidal variation of latitude at six International Latitude Observatories, 1900-1935

Table 11--Ocean effects on the M_2-term

Station	Oceanic effect	Distance from sea coast
	″ °	km
Mizusawa	0.0006 cos (2t-161)	50
Tschardjui	0.0003 cos (2t-292)	1300
Carloforte	0.0001 cos (2t-139)	0.17
Gaithersburg	0.0005 cos (2t-210)	66
Cincinnati	0.0003 cos (2t-169)	770
Ukiah	0.0012 cos (2t-245)	55

for the attraction of sea water, meridional displacement caused by tidal loading of sea water, and change of potential field by deformation of the Earth's surface. Table 11 gives the total effects of ocean tide for the M_2-term. These values show that the indirect effect of ocean tide is very small compared with the direct effect in case of latitude observations contrary to the case of tiltmetric observations. We could not explain why the tidal variation of latitude at Mizusawa and Gaithersburg are very much smaller compared with the other stations, but the work on this problem will be postponed for future investigation. Selecting the four stations, except Mizusawa and Gaithersburg, and applying the above calculated small correction of ocean tide, we obtained the pure direct values of tidal variations of latitude shown in Table 12.

Some points of these results may be discussed. Equation (6) gives L as an amplitude ratio of the observed value of tidal variation of latitude to the theoretical value (0.″0077)cos 2t) for a perfectly rigid Earth; hence L may be derived giving the values shown.

These results, may be interpreted in different ways. One explanation is that the differences of L may be almost attributed to the length of epoch analyzed. From such a point of view, one treatment is to take their weighted mean value, with length of epoch analyzed as a measure of weight. The mean value thus obtained is $L = 1 + k - \ell = 1.20 \pm 0.10$. Rewriting D, already obtained, for comparison, we have $D = 1 + k - h = 0.66 \pm 0.02$. For the purpose of obtaining accurate values of k, h, and ℓ without any assumption concerning the internal structure of the Earth, we need some other observations, such as gravimeter observations or determinations of Chandler's period. We hope to have an opportunity to discuss this point fully in later investigations.

On the other hand, the four stations cited (Table 12) may be divided into two groups: Continental (Tschardjui and Cincinnati) and coastal (Carloforte and Ukiah). H. Kimura had an opinion that there exists a certain essential difference between the continental and coastal stations in various points in the phenomenon of latitude variation proper. Hence, in this case, another treatment is to discuss separately the continental and coastal station. Taking the mean values, we have (1) coastal stations, L = 1.07; (2) continental stations, L = 1.49. In the present state, we cannot judge which is the true value of L in the problem of Earth tide or whether both are true values.

Table 12--Direct values of tidal variations of latitude and of L

Station	M$_2$	L	Years in analysis
	″ h		
Carloforte	0.0083 cos (2t-11.8)	1.08 ± 0.06	35
Ukiah	0.0082 cos (2t- 0.3	1.06 ± 0.06	35
Tschardjui	0.0101 cos (2t-11.8)	1.31 ± 0.19	19
Cincinnati	0.0128 cos (2t-11.8)	1.66 ± 0.18	16

In conclusion, after the tidal analysis of observational data of latitude variation for epochs of 35 years at six stations of the International Latitude Service, we could ascertain some essential points on this phenomenon, but could not obtain any decisive value of L which is most important in considering this aspect of the problem of Earth tide. We have found:

(1) Oceanic effects for tidal variation of latitude are very small, hence the angular phase of the analyzed results are all nearly equal to the theoretical.

(2) The lack of agreement in values obtained from the observational data of different epochs at the same station should not be attributed to analyzing methods or error of calculation, but mainly to the epoch or values of data of that epoch itself. Its real reason remains unexplained, but it is a very interesting problem in astronomy and geophysics.

(3) We should adopt in calculation the data of a longer period of observation than hitherto considered sufficient, to make the obtainable final value of tidal variation insensible to unknown disturbances.

(4) Of the different values for each station which are theoretically considered to have the same value, we are uncertain whether or not to attribute this discrepancy to the peculiarity of the station or to the systematic errors introduced in calculation, depending on time of observation or, in other words, on the number of star pairs analyzed.

(5) We obtained the values of L from two different standpoints. (a) L = 1.20 is the mean value for the four stations used. (b) L continental = 1.49 and L coastal = 1.07 are found by discriminating between the continental and coastal stations. The problem of L will be postponed for future investigation.

CHAPTER V

Time variation of the elasticity of the crust of the Earth

We have often experienced severe earthquakes of large scale originating at the bottom of the sea along the Pacific coast of Japan. As one of the plans to observe the correlated phenomena, if any, between those earthquakes and the crustal movement of the Earth surface, the fused silica tiltmeters were set up at Makimine in Kyûshû Province. Makimine is situated in the central part of the east coast of Kyûshû Province and is 28 km distant from the Hyuganada, one of the active centers of large sea earthquakes. The observation room is 165 m below the ground surface in the adit of the Makimine Copper Mine (λ = 131° 24′E, ϕ = 32° 38′N). The country rock is clay slate of the Chichibu Palaeozoic system, and the main ore is pyrrhotite containing copper. The observations were started at the beginning of 1942 and are still in progress. Only ten days in total, during the continuous recording of four years, were interrupted by the accidents to recording devices, or stoppage of electric power. The main parts of the silica pendulums were never accidentally disturbed during these four years.

The observation room is adequate for the observing of minute movements of the crust, such as Earth tides, abrupt tilting of small amount, and other minute changes. Here no trace of tilting of diurnal and annual period of meteorological and seasonal origin can be found, and the secular tilting is very small compared with that of other observing rooms of considerable depth. The Makimine station, therefore, is considered to be very stable in the above stated sense and most suitable for the observation of the phenomena of Earth tides and other minute movements of the Earth crust. Two fused silica pendulums denoted as A and B were set orthogonally and the positive signs of both pendulums in the following analysis show the tilting of the ground in the directions of N 24° W and N 66° E, respectively. The sensitivities of both tiltmeters are 0.″0223/mm for A and 0.″0205/mm for B in a 184-cm optical distance with maximum fluctuations of six per cent and three per cent, respectively, during the whole period of observation.

Considering the long continuous series of records of Earth tides undisturbed by any change of non-tidal origin, it can be expected that in a normal case, quite good numerical values of component tides should be obtained by the analysis of one-month data. In fact the mean error of the numerical results obtained by the one-month analysis was ascertained not to exceed two per cent of the original values. Hence, we first tried to find the degree of reliability of one-month observation by comparing the analyzed values of each month with the mean value for one year of data. But contrary to our

Table 13--Continuous

Observer	Station	Position		Depth of the room	Distance from sea
		λ	φ		
		°	°	m	km
O. Hecker	Postdam, Germany	13.0E	52.4N	25	200
T. Shida	Kamigamo near Kyoto, Japan	135.7E	35.0N	3	50
W. Schweydar	Freiberg i. Sa., Germany	131.3E	50.9N	189	400
E. Nishimura	Minami Primary School in Beppu, Japan	131.5E	33.3N	2	0.4
E. Nishimura	Senrihama in Aso, Japan	131.5 E	32.9N	10	45

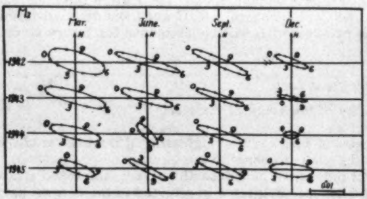

Fig. 7--Tilting diagram of the M_2-component

expectation, we observed a very large irregular fluctuation of the amplitudes of component tides in each month, large in extent and far beyond the error of observation and analysis. Hence, hereafter, we intend to detect the mode of fluctuation of values of component tides and clarify their meaning.

As a trial, the data of several months during the period of four years were harmonically analyzed and it was noticed that, as seen in Figure 7, the amplitudes of each month differ greatly from each other, but as to their phases, the fluctuations are rather small. Therefore, as a first step, the amplitude variations of component tides were studied in detail, especially concerning the M_2-component (lunar semi-diurnal tide). The data of each month using ten-day intervals were harmonically analyzed for the whole period. As to the O_1-component (lunar diurnal tide) the data of each three O_1-months were analyzed for the reason that the data of three months are needed to get moderate accuracy of results. Concerning the S_2-component (solar semi-diurnal tide) the data of six months were adopted to perfect the separation of the K_2-component (luni-solar semi-diurnal tide) whose

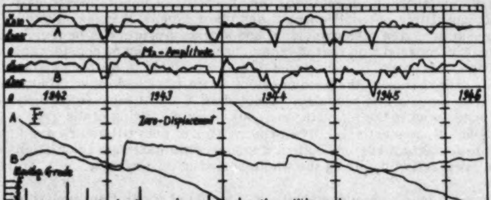

Fig. 8--Amplitude and zero-displacement at Makamine, with earthquake frequency shown tiltmeter observations

Type of tiltmeter	Sensitivity of tiltmeter	Epoch	Duration for comparison	Maximum fluctuation compared with mean value
	"/mm		yr	pct
v. Rebeur	0.04	1902-1909	2	±8
v. Rebeur	0.03	1910-1911	1/4	±9
Zöllner	0.02	1910-1915	1	±9
Silica	0.08	1937-1938	1/4	±3
Silica	0.007	1939-1940	1/4	±3

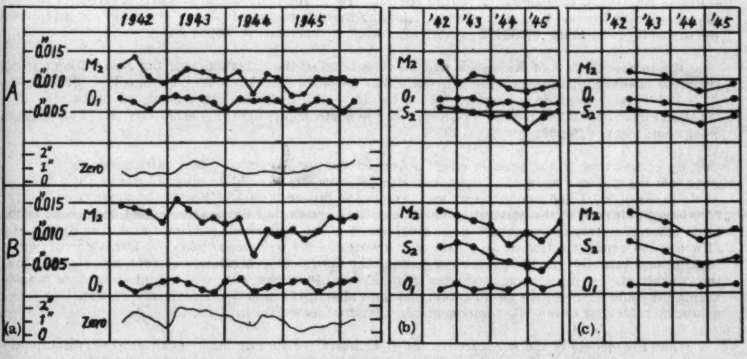

Fig. 9--Time variation of Earth tidal amplitude observed at Makimine; (a) three-months mean; (b) semi-annual mean; (c) annual mean

period is nearly equal to that of the S_2-component. As seen in the upper part of Figure 8, the total amplitudes of the M_2-component obtained by the analysis of successive ten days show large and irregular fluctuations, during the whole epoch, and their maximum range reaches more than 50 per cent of the mean value.

Some points on these enormous fluctuations of tidal amplitudes should be discussed. First, we must examine whether or not these fluctuations are apparently introduced by the process of harmonic analysis. To clarify this point, the comparisons of three-month values of M_2-components and O_1-component, and, moreover, six-month and annual values of M_2-, O_1-, S_2-components were made as seen in Figure 9. They show satisfactorily parallel change during the whole epoch, hence, it can be said that in reality the numerical values of Earth tides observed at Makimine change with time. The next problem investigated was whether or not these time variations are peculiar to the

Table 14--Fluctuations at Makimine

Pendulum	Duration for comparison, year		
	1/4	1/2	1
	pct	pct	pct
A	±32	±26	±13
B	±43	±27	±21

Makimine station. On this point we have, up to the present, no observations that are as long, continuous, and stable as those of Makimine. But for the sake of comparison several continuous observations of Earth tides with tiltmeters [HECKER and MEISSNER, 1911; SHIDA, 1912; and SCHWEYDAR, 1921] were selected, and the fluctuations of the tidal amplitude of the M_2-component are shown in Table 13. For comparison with Table 13 the maximum fluctuations of the M_2-component for A and B pendulum during 1942-1946 at Makimine are shown in Table 14. From these comparisons, it can be safely said that such an enormous fluctuation of tidal amplitudes with time, as observed at Makimine, cannot be observed, at least at the stations of Table 13.

Now returning to the problem concerning the Makimine station itself, a more detailed calculation on Earth tides observed at Makimine is necessary. In a normal case, the observed tidal variations of the Earth consist of two parts; (1) the direct effects of the Sun and Moon, and (2) their indirect effects through ocean tides. For the station near the sea the latter effect is generally larger than the former, and the main part of the latter effect is the bending of the crust caused by the variation of tidal loading of oceanic water. Therefore, as stated earlier, if we can extract the pure oceanic effect from the observed data, the elasticity of the crust of the Earth be calculated by comparison of the oceanic term observed with the attraction effect of tidal water calculated from the practical tidal chart of the neighboring oceans. For the purpose of estimation we selected, for an example, the numerical value of the M_2- component obtained from the analysis of data in September 1943. The cosine-form of this value corrected for the mean inclination of the Moon's orbit (Jm) and referred to the Makimine meridian is as follows: north-south component, 0."00287 cos (2t - 170°); East-west component, 0."00700 cos (2t - 10°). Next the calculated direct effect of the M_2-component for the Makimine station, adopting the diminishing factor of D = 0.661, is as follows: north-south component, 0."00475 cos 2t; east-west component, 0."00800 sin 2t.

The indirect effect of ocean tide will be calculated by the subtraction of the above direct effect from the observed data, hence the indirect effect becomes: North-south component, 0."00759 cos (2t - 176°); east-west component, 0."01025 cos (2t - 312°). On the other hand, the calculated attraction effect of tidal water is as follows: north-south component, 0."00377 cos (2t - 181°); east-west component, 0."00389 cos (2t - 303°).

By comparison of the oceanic effect observed and the attraction effect calculated, we see at once that the tilting diagrams of these two effects show similar oblate ellipses whose ratio of scale is about 2.3:1 and whose major axes are in the direction of S50°E with the lunar time of five hours referred to the Makimine meridian. This shows that the oceanic effect concerned is to be attributed mainly to the action of the tidal water of the Hyûganada, and moreover the principal effect is the bending action of tidal load of sea water in the Hyûganada upon the Earth's crust near Makimine. Actually, for the purpose of judging the degree of contribution of each sea region upon the oceanic effect, the surrounding seas of the Kyûshû Province were divided into four quadrants taking the Makimine station as a center, and the calculated attraction effects of sea water at each quadrant referring to the M_2-component are tabulated as shown in Table 15.

Thus the effects of the sea water of the Hyûganada and its adjoining ocean are overwhelmingly ponderous, especially in the bending effect, which becomes very significant for short distances, so that the Earth tidal values observed at Makimine will be intimately correlated to the action of sea water of the Hyûganada. On the other hand, it was ascertained that the ocean tidal height of the M_2-component observed with the tide gage at Hosozima facing the Hyûganada shows less than one per cent fluctuation according to the monthly analysis. Furthermore, under the reasonable assumption that the direct effect of the Sun and Moon is ever constant, we should conclude that time variation of Earth-tide amplitudes observed at Makimine is mainly introduced by the action of sea water of the Hyûganada, especially by the bending action of their tidal load. The amount of bending action is related to the mass of load, distance of action, and the elasticity of the crust. Therefore, under constant load and distance, the only variable element left is the elasticity of the crust or, in other words, the change of state of matter in the crust. What does the change of state of matter in the crust which gives 30 per cent fluctuation on the value of effective elasticity of the crust mean? It is difficult to give at present the correct answer. But setting the problem of its nature aside, if the apparent phenomenon that the degree of crustal bending varies with time by constant tidal loading is real, it should be expected that the secular change of inclination of the ground, that is, Table 15--Calculated attraction effects of the sea at Makimine

Area	Arc distance	S		W	
		cos 2t	sin 2t	cos 2t	sin 2t
	°	"	"	"	"
SE Quadrant	0.2 - 1.4	- 0.00121	0	+ 0.00157	0
Hyûganada	1.2 - 4.4	- 0.00104	- 0.00023	+ 0.00105	- 0.00009
SW Quadrant	1.2 - 4.4	- 0.00049	- 0.00047	- 0.00061	- 0.00183
NW Quadrant	0.7 - 4.4	- 0.00004	+ 0.00035	- 0.00002	- 0.00050
NE Quadrant	0.7 - 4.4	+ 0.00033	+ 0.00081	+ 0.00038	+ 0.00043
	4.4 - 44.4	- 0.00129	- 0.00097	+ 0.00015	- 0.00097
Sum		- 0.00374	- 0.00051	+ 0.00252	- 0.00296

the so-called "zero displacement" of the tiltmeter will also show such time variation with much larger amplitude than that in the case of tidal loading.

Generally speaking, the zero-displacements of the tiltmeter are largely disturbed by various effects. In case of observations in shallow-seated rooms, the meteorological effects, such as those of sunshine, rainfall, and change of barometric pressure, are most predominating. Even if the observing room is selected at considerable depth below the ground surface, the stability of the surrounding ground plays an important role in the problem of zero-displacement. Tiltmetric observation, for example, at Prǐbram HECKER, [1914] was unsuccessful in spite of its deep-seated room of 1110-m depth, because of the circumstance of excessively large displacements of the daily zero-position of the tiltmeter. Among the long and continuous tiltmetric observations in deep-seated rooms, those by SCHWEYDAR [1921] at Freiberg during 1910-1915, in the observing room of 189-m depth are considered to be the most excellent and stable hitherto obtained. But even there, the daily variation of meteorological origin (20 per cent of the M_2-component) and annual variation of seasonal origin (60 times the M_2-component) were observed and the zero-displacement amounted to about 80" and 130" during five years respectively for two orthogonally-set pendulums Returning to the case of Makimine, there is no trace of diurnal variation of meteorological origin and seasonal variation, and the secular variation for the A pendulum amounted to 12" during four years, pointing almost linearly in the same direction as is seen in the lower part of Figure 8. On the other hand, the B pendulum showed no secular variation; but irregular fluctuations were obsefved as is seen in Figure 8. Concerning the zero-shift to one direction of the A pendulum, it is considered not to be a real secular tilting of the ground; because, if real, the sensitivity of the B pendulum should be altered about 14 per cent of its initial value during four years in the sense of steady increase, but practically the irregular fluctuation of sensitivity of the B pendulum amounted in maximum to only three per cent. The total change in four years was less than 0.1 per cent of its initial value. Therefore, as a first step, we subtracted the large linear zero-shift of the A pendulum from the observed zero displacement and taking the monthly mean value of both the A (reduced) and B (original) pendulums, the comparisons were made for the three-month value of the M_2- and O_1-component as seen in Figure 9a. If it is assumed that the zero displacements thus gained are mainly to be attributed to the action of sea-water load of the Hyûganada, the comparison of the curve of zero displacement with that of tidal amplitude ought to be made, judging from the directions of both pendulums, by reversing the curve of zero displacement in the case of the A pendulum, and not reversing in the case of the B pendulum. It is here considered that the increase of inclination of the ground towards the Hyûganada means an increase of amplitude of tidal variation. According to this method of comparison, it is noticed that the tendencies of both time variations (tidal amplitude and zero-shift) are very similar, expecially for the B pendulum. The ratio of range of fluctuation between zero-shift and tidal amplitude is about 300:1, and the total amplitude of the M_2-component of ocean tide in the Hyûganada is one meter, therefore, the effective depth of sea water of the Hyûganada in the above stated meaning is estimated to be about 300 m. This is a value comparable to the mean depth of sea water of the Hyûganada near the coast.

As a next step the correlations between these fluctuation of tidal amplitudes and the occurrence of earthquakes originated in the Hyûganada were examined. At the bottom of Figure 8, the occurrences of earthquakes of Hyûganada origin were expressed according to their seismic intensity, that is: (1) insensible to human feeling; (2) sensible to human feeling, but feeble or local; (3) weak; (4) somewhat distinct, and (5) strong. And for the individual earthquakes we could not notice any intimate relation with the fluctuation of tidal amplitudes. But of the seismic activity, which will be defined as the total number of earthquakes that occurred in groups during a certain period, taking their individual seismic intensities into consideration, we see somewhat regular correlation between the seismic activity and the annual variation of tidal amplitude. There exists a tendency that the seismic activity is nearly zero when the tidal amplitude is smallest as in 1945. That is, if it is expressed in terms of elasticity, when the effective elasticity of the crust near Makimine is large, the seismic activity in the Hyûganada region is very feeble.

In summary, a very unusual result was obtained from the analyzed data of tiltmetric observation at Makimine. The Earth-tide amplitude observed at Makimine showed a notable fluctuation with time. This phenomenon is considered to be peculiar to the Makimine station. As a result of the various examinations, we can state that this fluctuation is not introduced by error in the analysis, but is intimately related to the bending motion of the Earth's crust by tidal load of sea water in the Hyûganada. As a natural consequence of the results, the state of matter in the crust near Makimine is considered to vary with time to a considerable extent, and though the nature of this variation is beyond our comprehension, we can at any rate watch the change of state of the crust continuously by such a method of tiltmetric observation.

In conclusion the writer expresses his cordial thanks to K. Sassa for his expert adivce through out this study, and to W. Inoue for his kind instructions on the data of earthquakes of the Hyûganada origin, and to K. Muto for his kind instructions on the data of the tide-gage at Hosozima. The writer also expresses his heartfelt thanks to K. Mukai and S. Sone (members of the Makimine Mining Corporation) for their continuous observations of long duration, and to S. Kobayashi for her diligent aid in the calculation of multitudinous data. The writer is greatly indebted to the Makimine Mining Corporation for its kind loan of the observation room and assistance given, and to the Society for the Advancement of National Arts and Sciences of Japan, whose grant and aid have made this study possible.

CHAPTER VI

Elastic anisotropy of the Earth's upper mantle (preliminary)

First of all, it must be mentioned that the subject of the following investigation does not involv direct observations, but is a possible interpretation of the characteristics derived from the observational data in the course of their analytical treatment. Therefore, more observations and analyses of different kinds are needed to ascertain the reality of the phenomenon.

Now, as described in Chapter I, we could separate the direct and indirect effects of the Sun and Moon in the analysis of tiltmetric observation by the difference method. We were able to cancel the unknown direct effects and treat the problem as one of pure indirect effects, the two being inseparably combined in an ordinary treatment of observational data at one station. In that way, we had the equation $m\ (r) = 12.6/(r + 3.0)$, which is the proportionality constant between the bending effect and the attraction component of tidal sea water, where r denotes the arc distance expressed in degrees between the particular sea region and the observing station. And, as the effective rigidity of the upper mantle of the Earth, the value of $\mu_e = 6.2 \times 10^{11}$ dynes/cm^2 was obtained. In that process of treatment, it should be mentioned that the values of m (r) and μ_e were determined in perfect independence of the direct effect, that is, D, the so-called diminishing factor. In the present investigation, for the purpose of testing the quality of the difference method, we tried to determine D from the observational data at each station, assuming the value of m (r) as a known quantity. There are, however, many uncertainties, as we have indicated, in analytical treatment of the observational data at one station near the sea; namely, inaccurate tidal height and phase of the open sea, non-applicability of Boussinesq's solution to far-distant points (effect of the Earth's curvature), and inadequacy of extension of (3) for m (r) of the upper mantle to the whole mantle. But the effects of the latter two uncertainties are comparatively small, and the error of the first effect is roughly estimated, in the mean, to be about ten per cent of total amount. It should therefore, be remembered that the following discussion involves uncertainty of that order of magnitude.

The observation of Earth tides were made, as already described, with the fused silica tiltmeters at two stations, Kamigamo Geophysical Observatory and Aso Volcanological Laboratory (see Fig. 10). Kamigamo ($\lambda = 135° 42'$E, $\phi = 35° 02'$N) is situated in the northern part of Kyoto City and its observing room is nine meters below the ground surface. Aso ($\lambda = 131° 00'$E, $\phi = 32° 53'$N) is in the district of Volcano Aso in Kyûshû Province, its observing room being 22 m deep. The sensitivities of tiltmeters at both stations were kept about 0.004"/mm at the optical distance of three meters. The epochs of the data analyzed were from January, 1939 to January,

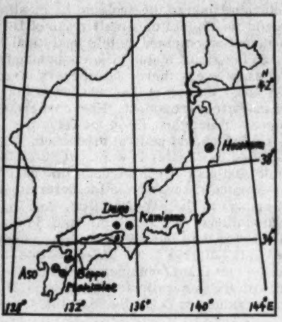

Fig. 10--Relative locations of stations discussed

1940 for Aso, and May to December, 1939, for Kamigamo. The observed values of tiltmeter and the calculated attraction components of sea water for the lunar semidiurnal component (M_2-tide), reduced to the mean inclination (Jm) of the Moon's orbital plane to the equatorial plane, and referred to the central meridian of 135°E, are as shown in Table 16.

Since the observed values are the combined results of both the direct and indirect effects, there will remain the direct effect after the process of vector subtraction of the indirect effect from the observed values. The indirect effect is composed of the attraction A of sea water in ocean tide, deformation B of the ground caused by tidal change of sea water acting as a surface load, and change C of gravitational field caused by deformation of ground by ocean tide. Therefore, in accordance with the reasoning described in Chapter I, the indirect effect S may be put in the form

$$S = A + B + C = A + mA - nA \quad \ldots\ldots\ (6)$$

where $m = 12.6/(r + 3.0)$ (r, in angular degrees) and $n = 0.5$ as shown in (3). Applying the above treatment, we have, as the direct effect, the numerical values shown in Table 16, and comparing with the theoretical values for a hypothetical perfectly rigid Earth, we may also obtain the D-values for these two stations.

Table 16--Data for computations of D at Aso and Kamigamo based on M_2-tide

Item	Component	M_2	
		Aso	Kamigamo
		″ °	″ °
Observed values	NS	0.00370 cos (2t - 189)	0.00339 cos (2t - 208)
	EW	0.00233 cos (2t - 314)	0.00569 cos (2t - 70)
Calculated attraction component of sea water	NS	0.00326 cos (2t - 198)	0.00297 cos (2t - 194)
	EW	0.00402 cos (2t - 286)	0.00191 cos (2t - 267)
Direct effect	NS	0.00372 cos (2t - 15)	0.00351 cos (2t + 3)
	EW	0.00921 cos (2t - 102)	0.00886 cos (2t - 85)
Theoretical value	NS	0.00720 cos (2t - 8)	0.00743 cos (2t + 1)
	EW	0.01326 cos (2t - 98)	0.01293 cos (2t - 89)
D	NS	0.52	0.47
	EW	0.69	0.68

On the other hand, as described in Chapter III, D was determined from the tiltmetric observation at Barim in Manchuria, where the indirect effect is very minute, and so D could be accurately discussed. In that case, the D showed numerical values with respect to the M_2-tide as follows: from NS component, 0.71; from EW component, 0.67; mean, 0.69. Comparing the computed values of D for the two stations Aso and Kamigamo with those obtained at Barim, favorable agreement occurs between them with respect to the EW-direction. But in the NS-direction, the values for the former two are much smaller than for the latter. What, then, does this discrepancy mean?

The elastic anisotropy of the Earth was first mentioned by HECKER and MEISSNER [1911] in the report on tiltmetric observations at Potsdam. And LOVE [1911] tried to explain the problem theoretically by gyroscopic rigidity caused the by Earth's rotation, but was unsuccessful in the matter of amounts. It was proved afterwards that the indirect effect of ocean tides, which Hecker neglected, would be sufficient to remove the apparent anisotropy of the Earth. And really the tiltmetric observations of MICHELSON [1919] and the writer, which were made at adequately conditioned observing stations where the indirect oceanic effects are very small, showed no anisotropy of the Earth. Michelson's observations were made in 1916-1917 at Williams Bay near Chicago and the writer's in 1941 at Barim in Manchuria. Table 17 compares the results.

Table 17--Values of D based on M_2-tide

Component	D	
	Michelson	Nishimura
NS	0.68	0.71
EW	0.69	0.67

Returning then to the problem in question, how should we explain the small value of D in the NS-direction obtained by this analytical treatment? So far as we maintain the standpoint that D is the same everywhere and in every direction, we must seek a source of discrepancy in the process of analytical treatment. Now concerning the process of analysis, (8) in the form $S = (1 + m - 0.5)A$ should be first discussed. This formula was determined by the difference method, as described in Chapter I as a most suitable method of relating the calculated attraction effect to the total indirect effect of ocean tide derived from the observational data. However, if we return to the original discussion in the determination of (8), it is obvious that this is mainly concerned with the amount in the EW-direction. And for the NS-direction, the difference method with respect to the two stations, Aso and Kamigamo, is inadequate because the amount treated in the NS-direction is 1/14 of that in the EW-direction, as seen in Figure 2. In other words, it can be safely said that the (8) is suitable in the EW-direction, but is questionable for the NS-direction. Again, reflecting on the fact that D obtained by the application of (8) in the EW-direction is satisfactorily consistent with the generally accepted value, it is considered that the reliability of the independently determined values of D and S are at the same time established to a certain degree.

Next, concerning the problem of discrepancy in the NS-direction, we assume tentatively that the value determined by (8) in the two directions differs a little. As a first step, assume that (8) in the NS-direction is of the form

$$S = A + mA - nA = A + f[12.6/(r + 3.0)]\,A - 0.5A \quad \ldots\ldots\ldots\ldots (9)$$

Calculation was made to find the most probable value of f that gives the normal value of D. Then the values obtained to give a value of $D = 0.68$ are: Aso, $f = 1.2$; Kamigamo, $f = 1.3$. Of course, there is room for argument over taking such a form for (8) assumed in (9), but in this discussion we adopted tentatively that form to explain the small value of D in the NS-direction. The fact that f is 1.2 or 1.3 means that the elasticity of the upper mantle of the Earth in the NS-direction is 20 or 30 per cent smaller than that in the EW-direction, because m (r) is inversely proportional to the elasticity of the material, and the bending effect, $B = m\,(r)\,A$, in this case mainly depends upon the elastic property of the upper mantle. The above reasoning will possibly be accepted when we consider that the wave length of deformation of the Earth by the direct effect (D-value) of the Sun and Moon is very large and really half the circumference of the whole Earth (for M_2-tide). Hence, a small irregularity of the crust and upper mantle of the Earth cannot be detected in determination of D. But on the contrary, the indirect effect, especially the bending effect $[m(r)A]$ rapidly increases with short distance between the actioning load of sea water and the observing station. Therefore, the bending effect is very sensitive to the small irregularity of property and structure of the material composing the layer of thickness comparable with the distance of oceanic action, as already discussed in Chapter II.

In addition to the two stations previously used, two other stations, Ikuno and Hosokura, were selected (see Fig. 10). Ikuno ($\lambda = 134°.8$ E, $\phi = 35°.2$N) and Hosokura ($\lambda = 140°.9$ E, $\phi = 38°.8$ N) are both copper mines and the observing rooms are situated in the adits, about 800 m and 160 m below the ground surface respectively. The sensitivities of the fused silica tiltmeters were kept to about 0.010″/mm and 0.007″/mm at about two meters optical distance, respectively. The observations were begun in June 1943 for Ikuno and in February 1944 for Hosokura, but the observations were interrupted by various accidents; hence, a good continuous record of long duration has not yet been obtained for both stations. Therefore, such a phenomenon of time variation of tidal amplitudes as observed at Makimine, which was discussed in Chapter V is not yet ascertained at these stations. Consequently, detailed investigation will be postponed until long continuous recording is accomplished. At present, only one month of data at both stations are selected and on the basis of these data the possibility of anisotropy will be discussed. Observed values and calculated attraction of ocean tides with respect to M_2-tide, corrected for Jm and referred to 135° E meridian, are shown in Table 18.

Applying (8) and subtracting the indirect effect from the observed value, we get the direct effect. And comparing them with the theoretical values, we obtain D as shown in Table 18. Thus, again in the case of the two stations Ikuno and Hosokura, the elasticity of the upper mantle shows smaller value in the NS-direction than in the EW-direction. Following the same method described above, f in (9) and D are in the NS-direction: Ikuno, $f = 1.2$, $D = 0.66$; Hosokura, $f = 1.3$, $D = 0.64$. A summary of the results for the four stations is shown in Table 19.

Table 18--Data for computation of D at Ikuno and Hosokura based on M_2-tide

Item	Component	Ikuno (August 1943)	Hosokura (November 1944)
		″ °	″ °
Observed value	NS	0.00240 cos (2t - 264)	0.00206 cos (2t - 349)
	EW	0.00458 cos (2t - 88)	0.00252 cos (2t - 57)
Calculated attraction	NS	0.00257 cos (2t - 207)	0.00115 cos (2t - 154)
	EW	0.00185 cos (2t - 266)	0.00268 cos (2t - 242)
Direct effect	NS	0.00422 cos (2t - 355)	0.00438 cos (2t - 332)
	EW	0.00789 cos (2t - 97)	0.00802 cos (2t - 70)
Theoretical value	NS	0.00743 cos 2t	0.00768 cos (2t - 349)
	EW	0.01292 sin 2t	0.01232 cos (2t - 79)
D	NS	0.57	0.57
	EW	0.62	0.65

Table 19--Summary of f and D values

Direction	Aso		Kamigamo		Ikuno		Hosokura	
	f	D	f	D	f	D	f	D
NS	1.2	0.68	1.3	0.68	1.2	0.66	1.3	0.64
EW	1.0	0.69	1.0	0.68	1.0	0.62	1.0	0.65

Therefore, roughly speaking, the elasticity of the upper mantle of the Earth in the NS-direction is 20-30 per cent smaller than that in the EW-direction.

Concerning the above result, a short critique follows: As stated at the beginning of this Chapter, the anisotropic property of the elasticity of the upper mantle, which was proposed to explain the small value of D in the NS-direction, is not a direct conclusion of observation, but merely one way of thinking of or interpreting a characteristic which appeared in the course of analysis of observational data. For the purpose of ascertaining real existance of this sort of anisotropy, it is necessary to seek other direct proofs. The following procedure is considered, for example, to be an adequate and reliable method to be employed to confirm the anisotropic property of the upper mantle. For instance, by taking two suitably selected observing stations, in which case the difference values in the difference method are fairly large and of the same order of magnitude in both directions (NS and EW), an S-formula and the effective rigidity of the upper mantle should be accurately determined in both directions. For that purpose, the two stations Kamigamo and Hosokura are supposed to be a fit pair, but, at present, the observational data for Hosokura is unfortunately scanty and insufficient to apply the difference method and to determine an S-formula and the effective rigidity. It is expected that in the near future good data at Hosokura can be obtained and will serve the purpose of resolving the problem of anisotropy. On the other hand, we hope that further observations to prove or disprove the existence of this anisotropy will be made at different places of the world. It is also necessary to investigate this anisotropy from a quite different aspect, such as the propagation velocity of seismic waves.

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EXPERIMENTAL STUDY OF SURFACE WAVES IN SHOALING WATER

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Abstract--A series of experiments was performed to compare actual wave dimensions with those given by theory for waves moving over a sloping beach. The primary consideration was the change of the water surface elevation at a point with time. It was found that this "profile" corresponded with Rayleigh's extension of the theory of waves to shoal water conditions, except probably in the case of rapidly shoaling water (slopes steeper than 1:10) and near the breaker zone.

Introduction--The first comprehensive analysis of the phenomena of surface waves was published by GERSTNER [1802]. This was based upon the geometrical conditions that a wave profile is trochoidal in shape with the path of individual water particles being circles in "deep water" and ellipses in "shallow water" with their axes decreasing with depth according to an exponential law. This theory was extended later by Froude and RANKINE [1863]. Experiments showed that wave profiles were approximated by theory except near the breaking point; however, this theory necessitated rotation while it was found that the formation of waves was necessarily irrotational.

AIRY [1842] showed the dependence of wave velocity on length and water depth, thus indicating the way for extensive mathematical treatments by STOKES [1947] and RAYLEIGH [1877].

The solution given by Stokes satisfied all the geometrical conditions in water of uniform depth and, in addition, required irrotational motion. The most interesting features of this solution were the facts that the velocity depended to some extent upon wave height as well as length and water depth and that the orbital motion was open rather than closed, thus indicating a mass transport in the direction of wave propagation. Later experiments [MITCHIM, 1940; and O'BRIEN, 1942] showed both of these findings to be correct. Stokes never proved the convergence of his series; however, LEVI-CIVITA [1925] proved it convergent for deep water and STRUIK [1926] proved a similar series convergent for shallow water waves.

RAYLEIGH [1877] worked on the problem of the difference between the wave front and the group velocities (energy front) noticed by Stokes, and he concluded that the energy was transmitted by the wave group velocity.

In nature waves must eventually travel into shoaling water or impinge against a cliff, while the various wave theories were developed for water of constant depth; however, RAYLEIGH [1911] discussed the variation of wave height in shoaling water using the assumption that waves in each depth had the same characteristics as in water over a horizontal bottom of the same depth. The problem of waves in shoaling water has finally been attacked theoretically by LEWY [1946] and STOKER [1948].

The work presented in this paper was done to check the validity of Rayleigh's assumption.

Description of equipment--The experiments described below were performed in a wave channel located in the Fluid Mechanics Laboratory at the University of California. This channel consisted of a metal tank, one foot wide, three feet deep and sixty feet long. The waves were generated by a flapper, located at one end of the channel, which was moved back and forth by a crank and an eccentric attached to a large horizontal flywheel. This assembly was driven through a series of gears and belts by a 1/2 hp Varidrive Syncrogear Motor. The wave period could be varied from about 0.7 sec to 2.2 sec. The wave amplitude was varied by adjustment of the crank and eccentric assembly.

Two beaches were used in the experiments (see Fig. 1). The first was constructed of aluminum and so mounted in the channel that the slope could be varied easily. It was set first at the

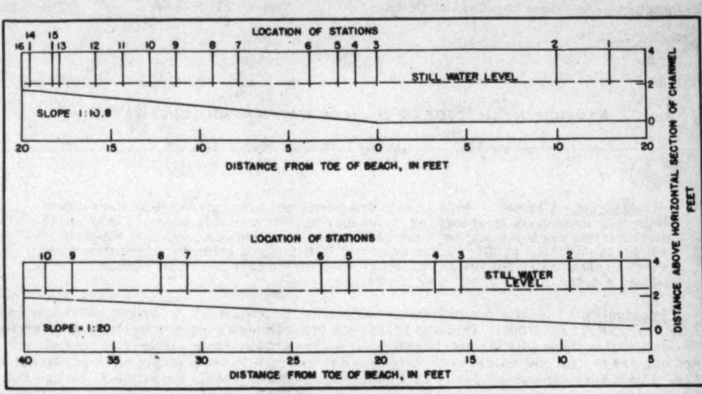

Fig. 1--Location of stations in wave channel where measurement of wave heights were made by resistance elements

steepest slope, which was 1:10.8, and a few runs were made on the smooth aluminum surface. The beach then was covered with a layer of mixed gravel and sand to reduce reflections and further runs were made. A plywood beach next was constructed with a slope of 1:20 for use in the final runs. Both beaches were stiffened so they would not flex under the variable pressure imposed on the beach with each passage of the waves. The beaches were mounted in the channel and sealed along the edges so that no flow of water would occur between the upper and lower sides of the beaches.

The wave dimensions were measured by using electrodes immersed in the water and connected in series with a recording oscillograph. Several basic types of electrode were experimented with before the final type was selected [WIEGEL, 1947]. This type, which consisted of a two-wire electrode, gave a straight-line calibration.

Experimental procedure--Measurements of wave height were taken at various points along the wave channel. It was found convenient to classify the points into two groups: those located in the section of the channel with the horizontal bottom, and those located along the sloping beach. Sixteen stations were used, the positions of which are shown in Figure 1. Note that wave heights were measured throughout the length of the wave channel and up to the point of breaking.

Each of the elements was individually mounted on point gages so that the element could be immersed to any chosen depth in the water. This feature aided in the calibration of the elements by setting them at various depths and operating the oscillograph. A calibration curve of depth against intercept on the oscillograph record was constructed for each element and used in determining wave heights from records taken following the calibration runs.

In an actual run the wave machine was started and adjusted for a desired period and wave height. It was found that for the desired periods the wave machine operated most effectively when the wave height was between 0.15 and 0.20 ft, which was fortunate as, with such heights, a one-to-one relationship existed between the actual height and the recorded height with a consequent increase in accuracy of measurement. Oscillograms were always made simultaneously of the fluctuations at two adjacent electrodes. This was done because the method of obtaining the velocity was to measure the time required for the wave to pass two electrodes which were located a known distance apart. The period was obtained from the same oscillogram as it was merely the time for two waves to pass a fixed point. The heights were obtained from these oscillograms by utilizing the calibration curves. The wave length then was determined from the product of velocity and period.

The main reason for using the electrode system of measurement, aside from obtaining the elevation-time relationship, was to measure the velocity and wave length over a shorter interval than could be done by actually noting the distance between crests by means of point gages. At first it was thought possible to set the electrodes only a few inches apart, but because of practical limitations in making measurements on the oscillograms it was found necessary to locate the electrodes about 1.5 ft apart.

The first runs were with all six of the oscillograph galvanometers in operation simultaneously as only the length, velocity, and period of the waves were desired. However, the later runs were made with only two channels of the oscillograph in operation at any instant so that the height could be obtained more accurately.

Discussion of results--Although the wave generator could not form waves with true theoretical orbital motion immediately in front of the wave flap, previous experiments in which orbital paths were determined photographically had shown that the waves assumed a theoretically correct motion after traveling a relatively short distance [BOUCHER, 1947]. To determine, in the present studies, whether the wave shape was trochoidal, measurements were made on the change of water surface elevation with time when a wave passed a given point. The elevation-time curve would not necessarily be (and probably could not be) the same as the composite wave profile over a sloping bottom, but it should be the same shape as the theoretical wave in water of that particular constant depth.

The trochoidal wave shape, considered in dimensionless form, can be expressed as follows [GAILLARD, 1904]

$$x/L = \mathrm{rad}[\cos^{-1}(2y/H)]/2\pi - \left\{(H \sin[\cos^{-1}(2y/H)]\right\}/2L \tanh(2\pi D/L$$

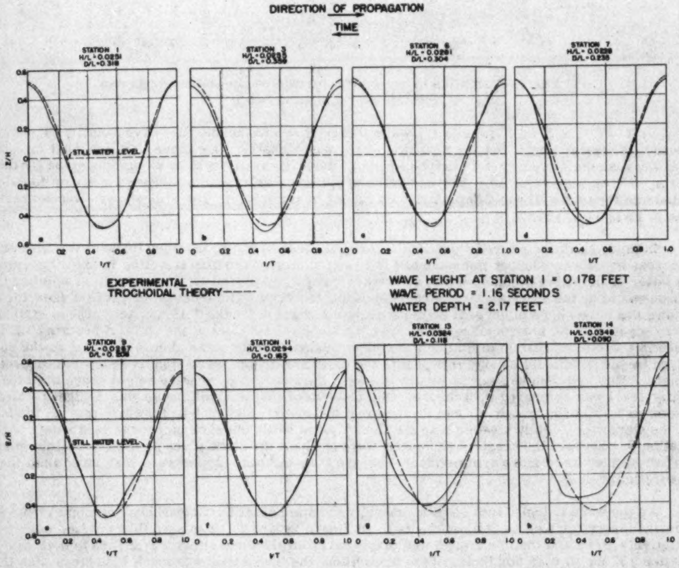

Fig. 2--Comparison between experimental and theoretical shape of waves moving over a beach with a slope of 1:10.8

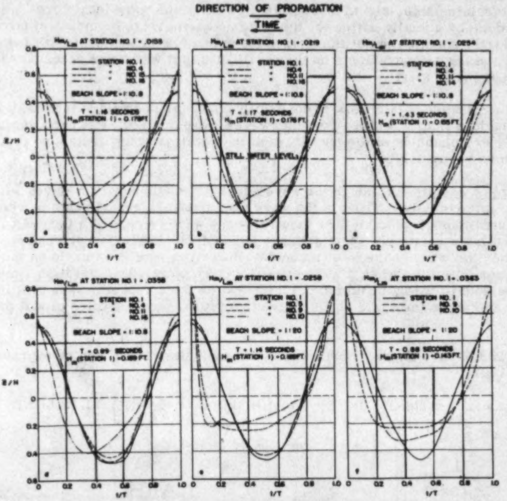

Fig. 3--Variation in wave form for waves of various steepness moving over a sloping beach

and, as L = CT, x/L = t/T (where L = wave length, H = wave height, C = wave velocity, T = wave period, D = water depth, and x = a portion of the wave length). The above equation must be modified because the wave center is displaced above the still-water level by an amount given by the term, $\pi H/4L \tanh(2\pi D/L)$. For any value of y/H the corresponding value of x/L can be calculated and the true y/H value determined by adding or subtracting this still-water level correction factor [WIEGEL, 1949].

In studying the measured "profiles", a certain leeway had to be taken because the immediate interest was not in whether the wave had the same dimensions while traveling in shoaling water as in water of the same constant depth, but rather, for the measured D, T, H, and L, in whether the shape would be the same as for the corresponding trochoid. The first set of profiles (see Fig. 2) shows the wave shape at several stations along the channel for the 1:10.8 beach. These stations were located in the horizontal section, over the sloping beach and at the point of breaking. It is seen in these plots that up to Station 9 the experimental curves were almost identical to the curve given by the trochiodal theory, except that the crest and trough were slightly below the theoretical points. This also can be seen in Figure 8, where the percentage of wave height above the still-water level was compared with theory. One other item that was noticeable was a slight lack of symmetry about the trough. It can be seen that the wave tended to lean forward; that is, the face of the wave was slightly steeper than the theory value while the back slope was less steep. At Station 9, however, the crests and troughs were at about the same place as the theoretical curve, and while they were more symmetrical, both the face and back slopes were less steep than the theoretical values.

As the wave traveled into shallow water, its shape changed considerably from that of the trochoidal theory (as well as Stokes' theory). In Figure 2g and 2h, it is seen that the face became relatively steep and that the trough was displaced from the theoretical t/T of 0.50 to 0.45 (for Station 13) and to 0.35 (for Station 14). In addition, the back slope was much less steep than the theoretical value near the trough, but increased near the crest because of the greater measured y/H. Several of the curves obtained during the run illustrated in Figure 2 are plotted together in Figure 3a for convenience in making comparisons.

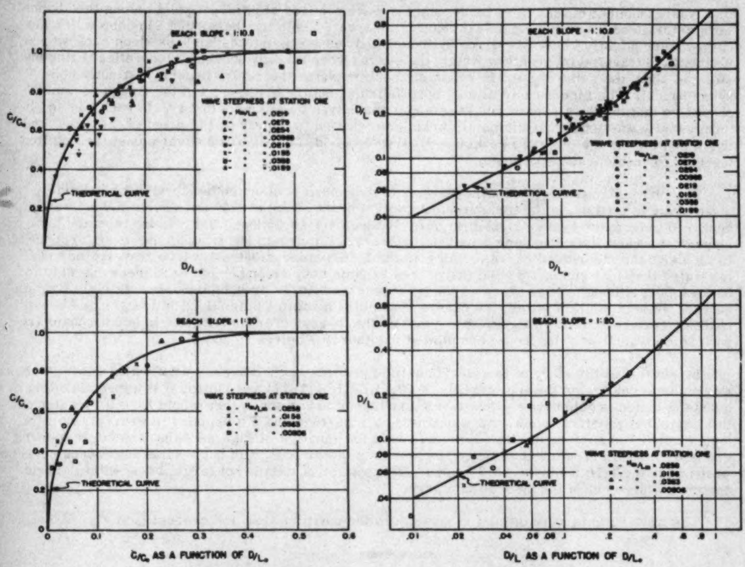

Fig. 4--Comparison between theoretical and measured wave characteristics

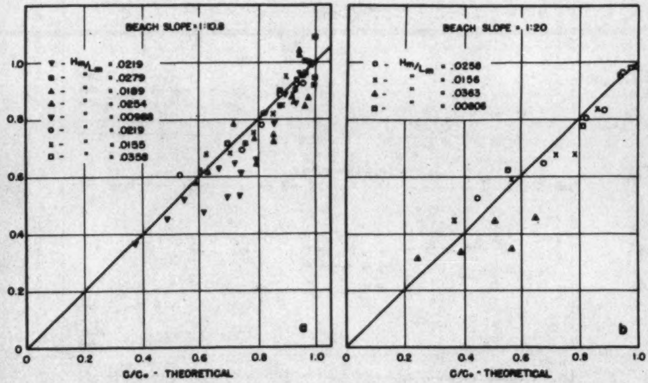

Fig. 5--Comparison between measured and theoretical values of C/C_O; theoretical values of C/C_O were computed from D/L_O values

A large number of runs were made, and the wave profiles at different points along the channel were determined. The same general observations were made for these runs. In deep water the curves were slightly below that given by theory and had steeper faces but less steep back slopes. As the wave traveled into shallow water, the curves were slightly above the theoretical value but still had about the same shape. In relatively shallow water the curves became distinctly nonsymmetrical. The face became almost perpendicular before breaking, and the trough became U-shaped so that the back slope was also steeper than given by the wave theory. In each run an element usually was located at the point where foam started to appear on the crest of the wave, and this was taken as the point of breaking. Figures 3a to 3d, inclusive, show various curves plotted together for comparative purposes.

The above discussion of results pertains to measurements on the beach with the relatively steep slope of 1:10.8. In the measurements with a beach slope of 1:20 the waves in deep water appeared to be more symmetrical than observed on the 1:10.8 slope, and both the face and back slope of the wave were the same as given by theory. The crests and troughs, however, appeared to lie above the theoretical curves. As with the 1:10.8 slope, when the waves traveled into shallow water they lost symmetry, and their faces became very steep compared to theory as did the back slopes, although not as steep as the front faces; while the troughs became relatively flat. In general, in very shallow water, the curves resembled a capital letter U. The troughs and crests of these were considerably higher than given by the theory. For convenience in making comparisons some typical profiles have been plotted together in Figures 3e and 3f.

In addition to the study of the elevation-time profiles of the waves, as discussed above, a check was made to determine the applicability of RAYLEIGH'S [1877] assumption that waves traveling ove a sloping bottom would have the same relationships at any point as they would have if in water of that same, but constant, depth. As illustrated by Figures 4 and 5 it appears, in general, that the theory applies in such instances. However, upon examination of Figures 4, 5a, and 5b, it appears that both the wave length and velocity are less than theoretical, with but a small difference between waves over the 1:10.8 and the 1:20 slopes. The scatter of data is not large, being within -15 per cent to + 5 per cent of the theoretical curve.

The wave heights were plotted in relation to the position along the channel (see Fig. 6) and,

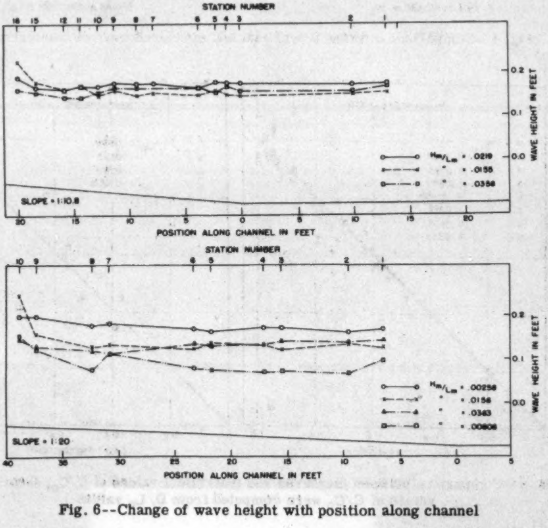

Fig. 6--Change of wave height with position along channel

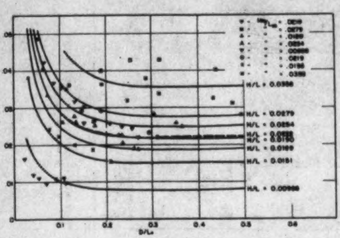

Fig. 7--Variation in wave steepness with position over a sloping beach

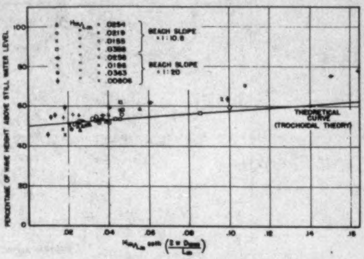

Fig. 8--Comparison between experimental and theoretical percentages of wave height above still-water level

as can be seen, there was considerable variation in height from point to point. Some of this variation was due, no doubt, to reflections which would exist on slopes as steep as those used. In addition, the error in measuring heights would be large compared with other measurements. In general, however, the height changes followed theory as can be seen in Figure 7. These plots show the change in wave steepness with decreasing depth. The location of the measured values of steepness usually was slightly above the theoretical curve.

The per cent of the wave height lying above the still-water level is shown in Figure 8. Again, in general, these data followed theory; however, for the beach slope 1:10.8 the deep water points were below the curve while for the 1:20 beach slope they were above the theory. For both beach slopes, the points near breaking were considerably higher than given by theory. The reasons for this condition were probably twofold. First, a hydrostatic head would have been formed near the breaker zone in a closed channel due to mass transport [MITCHIM, 1940; O'BRIEN, 1942; and UNIVERSITY OF CALIFORNIA, 1945]. Thus, the effective still-water level would be higher than that with no wave action, and, if this effect had been measured and included in the analysis, it would have made the points fall nearer to the theoretical curve. Secondly, reflections from the beach probably affected the results. In regard to the hydrostatic head built up shoreward of the breaker, it should be noted that somewhere in the tank there should be a slight depression to compensate for the hydrostatic head, and this depression could account for the per cent of wave height being below theory in the region seaward from the breaker.

Wave power should remain constant as a wave travels into shoaling water except for some small loss due to friction. This experimental results (see Fig. 9) show a variation with position. The fluctuation could be explained by the variation of wave height with location. However, the tendency for a rapid rise in indicated power as the wave nears the breaking point can be accounted for only by the change in wave shape; that is, the wave becomes higher than theoretical while at the same time, it becomes less full in its cross section (see Fig. 2 and 3). Hence, the power computed by the theoretical equations, using the measured height, length, and velocity, must be greater than the actual power of the wave. This condition exists because the theoretical power equation was developed from an analysis of the theoretical wave shape which, at the point of breaking, is considerably different from the actual profile.

The last phase of this investigation was a comparison of the theoretical curve of mass transport as determined from the height measurements at Stations 1 and 2, and the computed transport, using the measured values of H and L along the channel. The results of this analysis are shown in Figure 10, where it is seen that the experiment and theory agree reasonably well.

Summary--In general, the shape of the elevation-time curves of a wave passing a particular point was found to be similar to the trochoidal theory as long as the wave was not in relatively shallow water (say $D/L > 0.15$). For the relatively steep beach (1:10.8) the wave was not symmetrical; it had a steeper face than back slope. Also, the position of the wave form was below the theoretical curve, and the face was steeper than given by theory while the back slope was less steep. With a beach slope of 1:20 the above differences between theory and experiment no longer existed, and the surface curve was almost exactly the same as the trochoid, except that it was slightly above the theoretical curve.

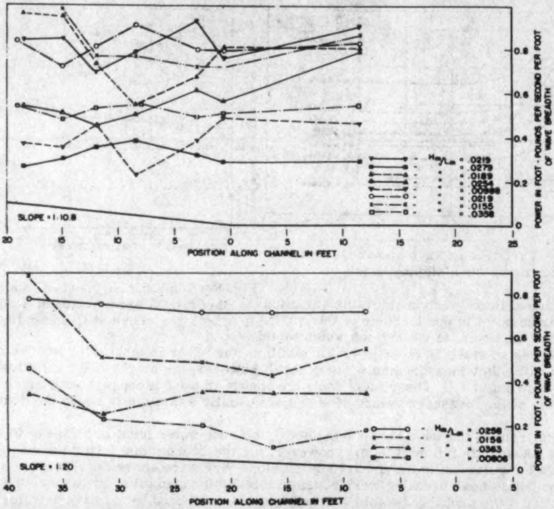

Fig. 9--Variation in wave power with position in channel

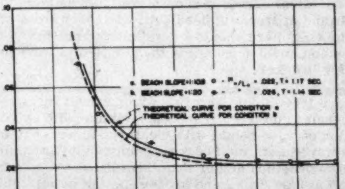

Fig. 10--Theoretical mass transport compared with values computed from experimental wave heights and lengths

When the wave had traveled into shallow water (over either slope), its shape varied greatly from that indicated by theory. The surface curve was above the theoretical surface curve. Both the face and the back slope became much steeper than in theory, and the wave was unsymmetrical, having a steeper face than back slope. In addition the trough moved toward the face side of the crest and broadened so that at breaking it was almost horizontal. This indicates that because the front of the wave is in slightly shallower water than the back part, it travels more slowly, and so the rear portion tends to push forward onto the front, steepening the wave and making a slightly higher crest and flatter trough.

In general, the assumption that the wave at any particular depth along a slope would have the same dimensions as it would in water of the depth over a horizontal bottom appeared to be valid, except that both the wave length and velocity were slightly less than the theoretical value over most of the shoaling section. The scatter of data was almost entirely within from -15 per cent to + 5 per cent of the theoretical values.

The change in wave height with decreasing depth followed theory except for certain variations. These variations in wave height along the channel were probably due to reflections from the steep beaches and to the accuracy required in height measurements. These variations in height were largely responsible for the apparently varying power along the channel. However, despite the fluctuations, there was no decrease in power as noted by STELZENMULLER [1941]; in fact, if the measurements just before breaking were to be neglected, the power would have remained fairly constant throughout the channel. The power, as computed from measurements at the stations near where the waves broke, was high because it was based on a theoretical wave shape; whereas, in reality, the wave was higher (thus indicating more power) but not as full in cross section, which would account for the difference in theoretically computed power.

Acknowledgments--A large part of the laboratory work accomplished in this investigation was part of an extensive study on waves and related problems conducted by the University of California for the Bureau of Ships of the U. S. Navy. The remainder of the study was done as a thesis submitted in partial satisfaction of the requirements for the degree of Master of Science in Mechanical Engineering at the University of California. The writer desires to express his appreciation for the valuable advice of J. W. Johnson, E. V. Laitone, and H. A. Einstein.

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RELATIONSHIPS BETWEEN WIND AND WAVES, ABBOTTS LAGOON, CALIFORNIA

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Abstract--A series of observations on wind-generated waves was made in Abbotts Lagoon, California, to define better the relationships between wind and wave characteristics on relatively small bodies of water of limited fetch. The experimental methods are described and a relationship between wave height and period as a function of fetch and wind speed is presented. Other information presented includes a relationship between wave steepness and wave age, wind gradients, and a typical frequency distribution of wave heights.

Introduction--The relationships between the characteristics of waves that are generated by wind in relatively small bodies of water are of considerable importance in many engineering problems. The general form of these relationships may be determined from theoretical considerations [SVERDRUP and MUNK, 1947] or from the model laws for the motion of a frictionless fluid under the action of gravity [JOHNSON, 1948]. In either case, numerical coefficients in such relationships must be determined from experimental observations. SVERDRUP and MUNK [1947] have determined the values of these coefficients from observations at sea and on relatively large lakes. Whether such relationships are applicable to relatively small lakes or protected bays where the fetch is limited has not been determined except for some preliminary observations at Clear Lake, California [JOHNSON, 1948]. The latter observations, however, were limited in value because the topography surrounding the lake influenced the wind pattern such that two-dimensional flow probably did not exist at the time the wind and wave observations were made. To supplement the Clear Lake data, a series of observations was taken at Abbotts Lagoon, California, where essentially two-dimensional flow occurs. The discussion to follow describes the experimental techniques used and the relationships between the wind and wave characteristics determined in this investigation.

Experimental equipment--Abbotts Lagoon is located on the Point Reyes peninsula, approximately 50 mi north of San Francisco (see Fig. 1). The lagoon is roughly rectangular in shape with a length of approximately 3100 ft, an average width of 1200 ft, and an average depth of about 30 ft. Relatively low topography surrounds the lagoon (see Fig. 2). It is separated from the open ocean by low sand dunes with a relatively narrow neck of water extending from the north end of the lagoon almost to the open ocean. In times of heavy surf action on the coast, waves have broken over the sand dunes, and sea water has entered the lagoon. During the tests described below, however, no ocean water entered the lagoon by wave action and the lagoon water was essentially fresh (average hydrometer reading was 1.008). The prevailing winds in this vicinity blow from the open ocean parallel to the long dimension of the lagoon. To give a definite limit to the fetch, a floating boom was constructed at the northern end of the lagoon (Fig. 1). The boom served to prevent the waves generated in the neck of water from entering the main body of the lagoon. The shoreline on all sides of the lagoon was a beach capable of absorbing all the waves generated in the lagoon; hence, no reflections were present to confuse a wave record.

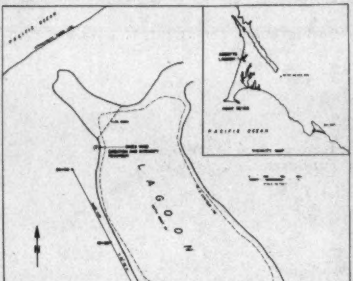

Fig. 1--Map of Abbotts Lagoon, California showing location of wind and wave recorders

The basic observations in this series of tests consisted of the speed, direction, and duration of the wind, and the height and period of the waves generated by these winds. For observations of wind direction two Dines recorders

Fig. 2--Aerial view of Abbotts Lagoon, California, showing the general character of the topography surrounding the lagoon

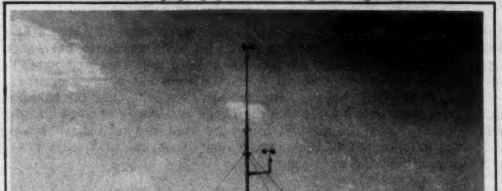

Fig. 3--View of float with anemometers for wind speed measurements at Abbotts Lagoon; top anemometer is eight meters above still water level

were installed at the two ends of the lagoon as shown in Figure 1. The records from these instruments were used primarily to obtain the wind direction and duration. The chart speed and response characteristics of the Dines recorders made them unsuitable for obtaining reliable wind speed data. To measure wind speed a bank of cup anemometers was installed at the edge of the water near the end of the fetch (southerly end of the lagoon) as shown in Figure 1. Anemometers were located at elevations of 4, 8, 12, and 16 ft above the still-water level in the lagoon. A plot of the vertical distribution of wind speed, however, indicated that a slight three-dimensional effect existed in the wind pattern in the vicinity of this anemometer station. Hence, another bank of anemometers was located on a float as shown in Figure 1. Five anemometers were provided on the float (see Fig. 3), with the top anemometer located at an elevation of 26.2 ft above still-water level. The other anemometers in the bank were located at the same elevations as those at the shore; that is, at elevations 4, 8, 12, and 16 ft. A fifth anemometer, also located at an elevation of 26.2 ft, was installed on the shore bank of anemometers at the time of construction of the float assembly.

Underwater cables from the anemometers on the float led to a central recorder house. The wind speeds for each bank of anemometers were recorded simultaneously on five-pen strip-chart recorders. Thus, each electrical contact in a particular anemometer was recorded as a "pip" on a single chart. From the measured chart speed the pips per unit time were determined and the wind speed then obtained from a calibration curve for the particular anemometer. All cup anemometers and the two Dines recorders were calibrated in the University of California wind tunnel before and after the observations were made at Abbotts Lagoon.

The wave recorder used in this investigation was of the pressure type; that is, pressure variations at a subsurface point were recorded on a strip chart from which wave height and periods could be obtained. Essentially, the instrument consisted of a submerged inverted air-filled plastic cup which communicated pressure variations through a tube to a bellows. The motion of the bellows was transmitted by a linkage to a pen bearing on the strip chart of a clock-driven recorder. Either one of two inverted cups could be used at any time, one cup being located at six inches below the still-water level for recording relatively small waves and the other being located at a depth of 18 inches for recording larger waves. The total water depth at the point of recording was six feet. The wave recorder was calibrated in the wave channel located in the Fluid Mechanics Laboratory at the University. It was calibrated both before and after the field tests, over the range of wave heights and periods encountered in the field. A calibration curve thus was prepared to convert subsurface pressures into surface wave heights.

Other data recorded during the progress of this study were wet and dry bulb air temperatures, water temperature, specific gravity of water, and a photographic record of the characteristics of the waves.

Experimental data--Observations of wind and waves were made at Abbotts Lagoon during the months of November and December in 1947 and 1948. Experimental data were obtained on 15 days during the period of observation. Except for the Dines recorders which operated continuously, the various recording equipment was operated only when the wind was blowing approximately along the long dimension of the lagoon. The records from the wave recorder and the banks of cup anemometers were analyzed only for those periods for which the Dines recorders showed the wind direction and speed to be relatively stable. That is, the duration of the wind was such that the waves reached the maximum height for the existing wind. Under these conditions, the maximum wave height was controlled by the fetch and not the wind duration.

The procedure in analyzing the anemometer records was first to mark the strip chart at five-minute intervals. At each such interval the number of pips occurring over a one-minute length of the chart were determined for each anemometer. The wind speed then was determined from the anemometer calibration charts. The wind direction was read directly from the charts from the Dines recorders located at the two ends of the lagoon. In all tests these two recorders gave the same wind direction, thus indicating that a two-dimensional flow condition prevailed over the lagoon. With the wind direction known, the fetch length was determined by scaling the distance from the wave recorder to the water's edge at the northern end of the lagoon (Fig. 1).

The charts from the wave recorder were analyzed for wave height and period at the same five-minute intervals as in the wind data analyses. A typical wave record is shown in Figure 4. For each five-minute interval the average wave period was obtained by counting a number of distinct waves and dividing by the elapsed time. The average wave height reported is the "significant wave" or the average of the highest one-third of all waves present [SVERDRUP and MUNK, 1947]. The procedure in arriving at this average was, first, to compute the total number of waves occurring in

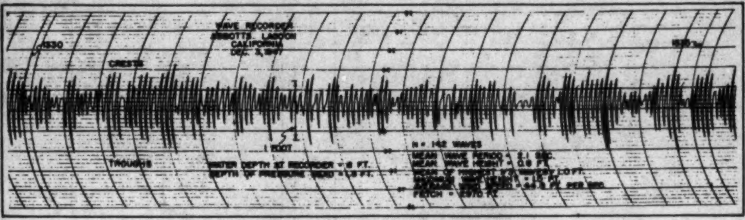

Fig. 4--Chart from Abbotts Lagoon wave recorder showing the variability of the height and period of waves with time

each five-minute interval. Taking one-third of this total number of waves, this number of the highest waves were checked on the chart, their individual heights (trough to crest) measured on the charts and determined from the wave recorder calibration curve, and the average of these wave heights then computed.

From the above analyses a compilation was made of fetch, average wind speed at various elevations above the water surface, and wave height and period as a function of time.

Summary of data—As has been shown previously [SVERDRUP and MUNK, 1947], the relationship between wind and wave characteristics can be shown best by dimensionless representations. For those conditions where the wave characteristics are controlled by fetch, and not wind duration, observed data on wave and wind characteristics should reduce to two diagrams, one representing wave height and the other representing wave period, according to the following functions

$$gH^2/U^2 = f'(gF/U^2)$$

and

$$gT/U = f''(gF/U^2)$$

where H is the significant wave height in feet, T is the wave period in seconds, F is the fetch in feet, U is the wind velocity in ft/sec and g is the acceleration of gravity in ft/sec^2.

The above parameters were calculated from the Abbotts Lagoon data and are shown in Figure 5. The wind speed, U, in these representations is the speed at the elevation of 26.2 ft above the water surface. Also shown on these figures are the curves of SVERDRUP and MUNK [1947] and the data of FLINSCH [1946] and STANTON, MARSHALL, and HOUGHTON [1932]. Examination of these curves shows that the Abbotts Lagoon data for wave height scatter around the Sverdrup-Munk curves; however, the data on wave period lie (see Fig. 5a) consistently above the Sverdrup-Munk curve. That the latter curve gives wave periods which are generally too low has been shown by ISAACS and SAVILLE [1949] in a comparison between recorded waves on the Pacific Coast with those forecast from the Sverdrup-Munk curves by the standard technique of wave forecasting [U. S. HYDROGRAPHIC OFFICE, 1945]. The data summarized in Figure 5 cover a range of conditions which previously was lacking entirely; that is, the above observations lie between the FLINSCH [1946] and STANTON, MARSHALL, and HOUGHTON [1932] data from flume studies and the SVERDRUP-MUNK [1947] data obtained primarily from observations in the open ocean. The Abbotts Lagoon data fall in a region more typical of small lakes and protected bays where the fetch is in the order of less than about ten miles and the wind speed is less than 50 mi/hr. For a given wind speed and fetch, these curves appear useful in estimating the height and period of the waves that can be expected under conditions where the wind duration is sufficient to raise waves to their maximum height.

Incidental information from the Abbotts Lagoon data that is of interest is the vertical gradient of wind speed over the lagoon shown in Figure 6. These wind gradients for various wind speeds were obtained from the bank of anemometers on the float in the lagoon. Also of interest is the relationship between wave age and wave steepness. The term wave age, as previously used by SVERDRUP and MUNK [1947], is the ratio of wave velocity to wind speed, C/U. Wave steepness is the ratio of wave height to wave length, H/L. A plot of these two terms is shown in Figure 7 with

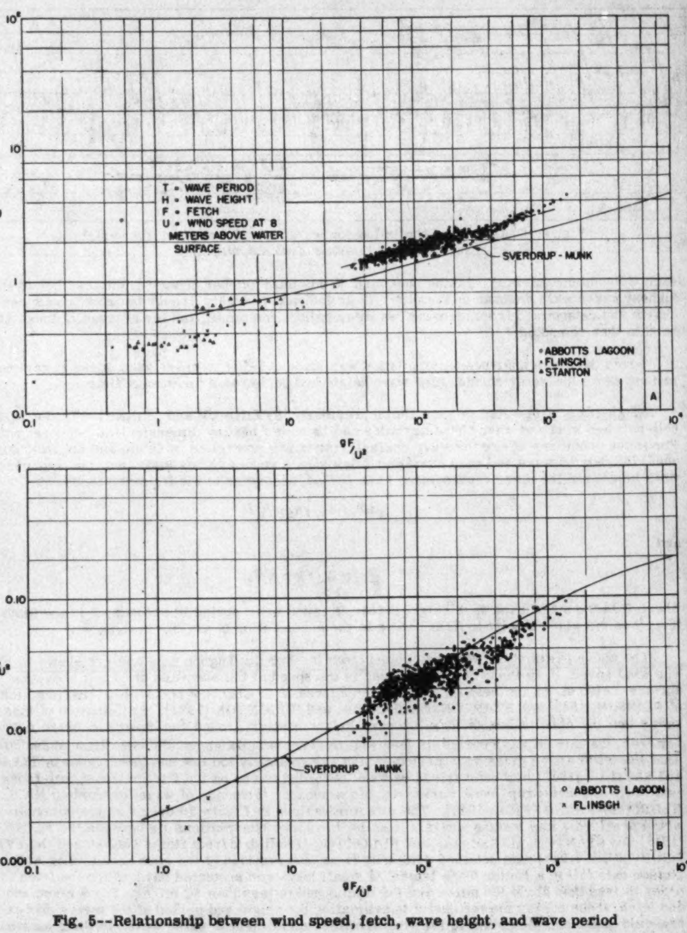

Fig. 5--Relationship between wind speed, fetch, wave height, and wave period

the curve of SVERDRUP and MUNK [1947] shown for comparative purposes. The fact that the Abbotts Lagoon data lie consistently below the Sverdrup-Munk curve to some extent, is caused by the fact that the wave period in the Abbotts Lagoon data is consistently larger than that which would be obtained from the Sverdrup-Munk curves for a given value of the term, gF/U^2. That is, the fact that both wave velocity, C, and wave length, L, are functions of wave period causes the computed values of wave age to be larger and the values of wave steepness to be smaller, respectively, than the Sverdrup-Munk data.

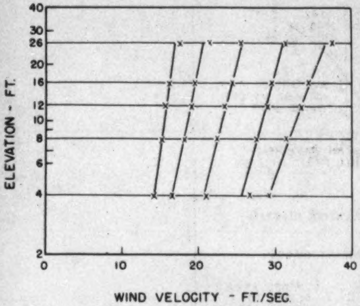

Fig. 6--Typical wind gradients at Abbotts Lagoon, California

In addition to the average wave height and period, the variability of waves is of considerable interest in many problems. A typical illustration showing the general character of the variability of waves in a generating area is shown in Figure 4 for a five-minute period of recording. To give some specific information on the variability of wave height, the height of each wave appearing in the record shown in Figure 4 was measured and tabulated. From this tabulation the following data were computed: (a) the mean wave period, (b) mean height of all waves appearing on the record, (c) the mean height of the highest one-third of the waves (the significant height as discussed above), and (d) the maximum wave height. The values of these various items are shown in Figure 4. Also computed from the above tabulation was a frequency distribution of relative wave heights. The wave heights in this instance are relative to the average of the highest one-third of the waves. A frequency diagram and a cumulative curve plotted from these data are shown in Figure 8. Assuming that this plot is typical of wind-generated waves, if the significant wave height is determined from Figure 5b for a given wind and fetch, the heights of the highest waves and of the mean waves can be computed, as well as the percentage of waves higher or lower than any given value. The heights of the lowest waves also can be determined; however, it should be recognized that the lower waves appearing on the record are a function of the depth at which the underwater unit of the recorder is placed. That is, the smaller waves that might have existed are eliminated from the records by hydrodynamic filtering.

Conclusions--The relationships between wind and wave characteristics, as developed from the Abbotts Lagoon data, give wave heights which agree with the curve of SVERDRUP and MUNK [1947]. The wave period, however, averages over 20 per cent higher than that given by SVERDRUP and MUNK [1947]. This relatively high period accounts, to some extent, for a relationship between wave age and wave steepness which differs considerably from that previously developed [SVERDRUP and MUNK 1947]. The relationships between wind and wave characteristics, as presented herein, appear useful in estimating wave conditions on lakes, reservoirs, and protected bays for those conditions which fall within the limits of the dimensionless term, gF/U^2, as considered in this investigation.

Acknowledgment--The work summarized in this paper was part of an extensive investigation on waves and surf made at the University of California in a project sponsored by the Bureau of Ships, United States Navy.

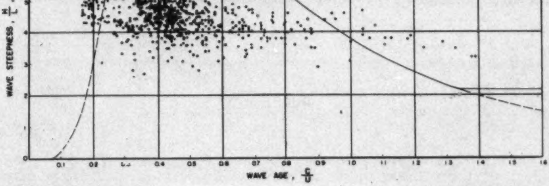

Fig. 7--Relationship between wave steepness and wave age

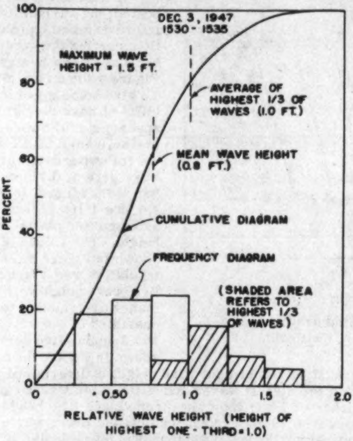

Fig. 8--Per cent of occurrence of relative wave height for typical data from Abbotts Lagoon, California

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THE WATER STRUCTURE IN THE BROWNSON DEEP

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(Contribution No. 495, Woods Hole Oceanographic Institution)

Abstract--Temperature and salinity observations were made in the Brownson Deep. The water below the sill depth is nearly homogeneous, showing a barely perceptible decrease in potential temperature and salinity with depth. The source of this deep water is shown to be modified Antarctic Bottom Water from the South Atlantic which enters the Deep from the east.

On a recent cruise of the research vessel Atlantis to the area north of Puerto Rico (February and March, 1949) serial observations of temperature and salinity were made at seven stations. Two of these stations, of which one consisted of a deep cast only, were of particular interest as they made possible an examination of the water in the Brownson Deep.

Figure 1 is a bathymetric chart of the area in question showing the station positions. The contours, in fathoms, were traced from U. S. Hydrographic Office Chart No. 5487. Although not considered accurate in every detail, the chart gives a good picture of the major bottom features of

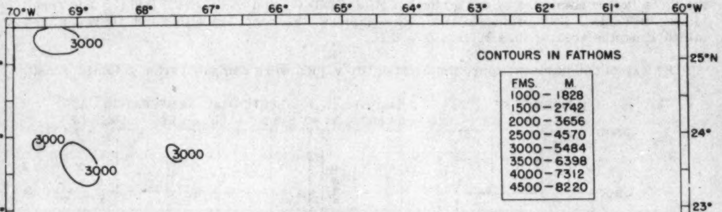
70°W
69°
68°
67°
66°
65°
64°
63°
62°
61°
60°W
25°N
24°
23°
22°
21°
20°
19°
18°N
25°
24°
23°
22°
21°
20°
19°
18°
70°
60°
CONTOURS IN FATHOMS
FMS. M.
1000 - 1828
1500 - 2742
2000 - 3656
2500 - 4570
3000 - 5484
3500 - 6398
4000 - 7312
4500 - 8220
● ATLANTIS STATIONS
3000
>3000
<3000
3500
4000
4500
2000
1500
1000
<1000
2500
#4754
#4749
#4751
#4753
#4750
#4752
#4755
SILVER BANK
HISPANIOLA

Fig. 1--Bathymetry of the Brownson Deep area with station positions the region. The bottom of the Brownson Deep, about 8250 m below the sea surface, extends 220 mi in an east-west direction. North of the Deep the average depth of bottom is 5500 m. To the eastward the trench shoals gradually until it merges with the main oceanic basin whose axis here trends toward the southeast between South America and the Mid-Atlantic Ridge. At the Equator this basin has a sill depth of about 4400 m. It then deepens to over 5000 m but shoals again to about 4300 m as it joins the Antarctic Ocean basin. Its eastern boundary, the Ridge, has an average sill depth of approximately 3500 m and is never deeper than 4000 m.

All observations below 3500 m, from the five stations extending beyond this depth, were combined into a composite plot showing temperature, potential temperature, and salinity versus depth (see Fig. 2). This procedure appears to be justified by the fact that only two of the points deviate from the mean curves by values greater than the observational errors.

Below 5600 m, the approximate sill depth of the Deep, the water is almost homogeneous, with a downward decrease of potential temperature of only 0.03°C per 1000 m and almost constant salinity. Perfectly homogeneous water would have constant potential temperature as well as constant salinity, and its stability would be zero. The fact that the potential temperature gradient in the Deep is real and not due to the composite nature of the graph is affirmed by its presence in each of the two deep casts plotted separately. If this gradient is the result of downward transfer of heat, by means of eddy diffusion, from the slightly warmer and more saline layer immediately above it, a salinity gradient would also be expected to occur in the deep water. On the assumption that the ratio of salinity gradients in the layers above and below the sill depth of the trench is the same as that of the corresponding potential temperature gradients, the salinity gradient in the deep water would be a decrease of less than 0.004^{0}/oo per 1000 m. This value is of the same order of magnitude as that shown by the mean salinity curve in Figure 2.

Salinities and temperatures were read off the mean curves at four points and the stability of the three layers computed by the method of HESSELBERG and SVERDRUP [1914]. The results are shown in Table 1. According to the above authors, the accuracy with which 10^8E can be determined at depths greater than 2000 m is ± 0.1.

By way of comparison, approximate stability values were computed from potential density

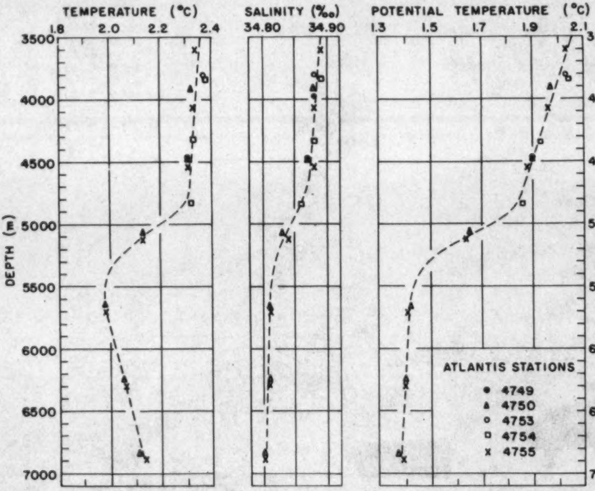

Fig. 2--Composite depth curves of T, S, and θ

Table 1--Stability above and below sill depth from method of HESSELBERG and SVERDRUP

Depth	Stability
m	10^8E
3600	
	+ 1.3
4800	
	+ 6.9
5600	
	+ 0.4
6900	

Table 2--Stability from potential density gradients

Depth	σ_θ	Approximate stability
m		$10^5 d\sigma_\theta/dz$
3600	27.90	
		- 0.8
4800	27.89	
		0.0
5600	27.89	
		- 0.8
6900	27.88	

gradients. These gradients were determined from the mean curve for σ_θ, a function of potential temperature and salinity at atmospheric pressure. The results of Table 2 were obtained. Values for σ_θ are rounded off to the second decimal place, commensurate with the accuracy of the data. The discrepancy between the true and approximate stability figures confirms the inadequacy of the potential density gradient method, already pointed out by HESSELBERG and SVERDRUP [1914], when dealing with typical deep water masses. It is clear that σ_t gradients would give even more erroneous stability values.

An examination of salinity sections across the Northern Equatorial Current and between Bermuda and Haiti and of a T-S correlation for bottom water between Bermuda and Chesapeake Bay [ISELIN, 1936, pp. 41, 51, 57] shows no salinities as low as those found in the Brownson Deep. The only source of water with the required characteristics as well as direct access to the Brownson Deep is Antarctic Bottom Water which has crossed the equator from the South Atlantic Ocean west of the Mid-Atlantic Ridge.

Data from the Meteor expedition to the South Atlantic and southern North Atlantic [WÜST 1932, 1936] show that the Antarctic Bottom Water spreads northward in a thin layer extending only a few hundred meters above the bottom in the deepest section of the western basin, always below 4000 m. This flow is characterized by a perceptible mean increase in its potential temperature and salinity as it crosses the equator and gradually mixes with southward-flowing North Atlantic Deep Water [SVERDRUP, JOHNSON, and FLEMING, 1942, p. 747]; its sluggish and intermittent character is indicated by the fact that the northward increase of temperature and salinity is quite irregular, with some stations having lower temperatures and salinities than those just south of them.

Figure 3 gives the potential temperature-salinity correlation below 1100 m for the seven Atlantis stations occupied on this cruise, with a mean curve drawn through the points. For comparison, Atlantis Station 1214, at latitude 25° 33′N, longitude 66° 45′W, typical of the Sargasso

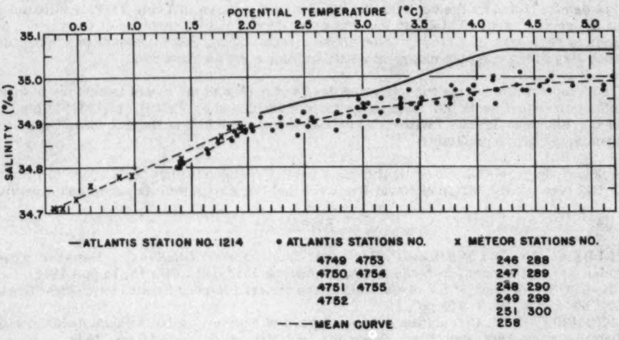

Fig. 3--Relationship of θ to S below 1100 m

Sea, is shown. The mean Sargasso Sea curve [ISELIN, 1936, p. 87] could not be used for this purpose because its conversion to potential temperature is impracticable and also because it does not extend below a temperature of 4°C. The third set of points in the figure are the deepest observations from a number of Meteor stations between latitudes 6° 11'S and 12° 38'N. These lie about a straight line whose extension merges with the lowest section of the curve for Station 1214, thus clearly illustrating the mixing, in varying proportions, of Antarctic Bottom Water and North Atlantic Deep Water.

It is noteworthy that the recent Atlantis observations have consistently lower salinities, at constant potential temperature, than either Station 1214 or the Meteor observations. These variations in the correlation between θ and S are described as differences in salinity rather than in temperature since the latter would imply variations of much greater magnitude than can reasonably be expected to exist at these depths. In the temperature range shown in Figure 3 the maximum difference in salinity between the data from the Brownson Deep area and from Station 1214 is 0.12 ^{o}/oo, while the mean difference is only about 0.05 ^{o}/oo. The Meteor observations average 0.02 ^{o}/oo higher than the corresponding ones from the Brownson Deep. These differences probably can be attributed to the time interval between the various sets of data, the Meteor observations having been made in 1927 and Station 1214 in 1932. The existence of such long-period fluctuations in the average characteristics of oceanic water masses is generally recognized even though their causes are not clearly understood. It is pertinent to point out that the deviations from the mean found in ISELIN'S [1936] Sargasso Sea T-S correlation are as great as the salinity differences mentioned above, even though the data used cover a span of only six years.

In considering the path by which Antarctic Bottom Water enters the Brownson Deep it is apposite to compare the present data with Snellius Station No. 262 made in the Mindanao Trench off the Philippine Islands to a depth of 10,000 m [VAN RIEL, 1934, p. 16, 17]. Except for the fact that its axis runs north-south rather than east-west, the general configuration and sill depth of the Trench are similar to those of the Brownson Deep. Van Riel shows that in a potential temperature section across the sill of the Trench the deep isotherms parallel the sea floor into the greatest depth. He considers this condition an indication of a horizontal flow along the bottom to which he gives the name "sill current."

On the basis of the Atlantis stations (see Fig. 2) no such sill current exists on the north side of the Brownson Deep. However, in view of the evidence from the Meteor data and the fact that at the eastern approach to the Deep the bottom drops off at a rate of less than one part in a hundred, it is probable that Antarctic Bottom Water enters the trench from the east along the bottom. This supposition cannot be verified since the two deep Atlantis casts were still 1400 m short of the greatest depth and no observations were obtained east of the 66th meridian.

At shallow and intermediate depths, flow patterns can frequently be determined by the method of isentropic analysis, the distribution of variables on a surface of constant potential density [MONTGOMERY, 1938]. The inapplicability of this technique to deep water studies, on the other hand, is demonstrated by the vertical distribution of σ_θ shown in Table 2. This situation is a result of the small degree of stability found at great depths and the inability of the various density functions to represent it correctly under those circumstances. The cause of this apparent discrepancy lies in the complex nature of a salt solution such as sea water.

The simplest alternative for analyzing deep water flow is the use of indicators in form of isopleths on vertical sections. That is the method employed by VAN RIEL [1934] in his investigation of the Mindanao Trench. With more data in the eastern half of the Brownson Deep a similar approach might prove profitable.

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TWO DIAGRAMS FOR FINDING BREAKER CHARACTERISTICS ALONG A STRAIGHT COAST

P. Groen and M. P. H. Weenink

Abstract--Two diagrams are offered, from which one may read directly the breaker characteristics for the case of sufficiently straight and parallel bottom contours, if the initial steepness and the crest angle in deep water are given.

If the conditions of sea and swell in the deep water off a coast are known, either from observation or from computation [SVERDRUP and MUNK, 1946a; 1947; SCRIPPS INSTITUTION OF OCEANOGRAPHY, 1948], it is possible to compute the characteristics of breakers and surf from the bottom topography of the shallow water, if the latter is known also [U. S. HYDROGRAPHIC OFFICE, 1944; SCRIPPS INSTITUTION OF OCEANOGRAPHY, 1945; SVERDRUP and MUNK, 1946b; MUNK and TRAYLOR, 1947]. In general it will be necessary to draw a refraction diagram for this purpose in order to find the effect of convergence or divergence on the wave height. In the case of sufficiently straight and parallel bottom contours this effect may, for any depth D, be found by

$$R = H_o'/H_o = \sqrt{\cos \alpha_o / \cos \alpha} \quad \ldots\ldots\ldots\ldots \quad (1)$$

in which H_o is the deep-water wave height, H_o' is a theoretical reduced wave height as found from the refractional divergence effect only, α_o is the known angle of incidence of the wave crests in the deep water, and α is the angle of incidence at depth D, as found from Snell's law

$$\sin \alpha / \sin \alpha_o = C\,(D)/C_o \quad \ldots\ldots\ldots\ldots \quad (2)$$

in which C_o and C (D) are the phase velocities in the deep water and at depth D, respectively. R^2 is the inverse of the factor by which the distance between two orthogonals has grown.

The refraction factor R, being a function of D/L_o and α_o, may, for values between $R = 0.50$ and $R = 0.95$, be read directly from the graphs of Plate II of U. S. HYDROGRAPHIC OFFICE [1944]. More generally, it may be computed from (1) and (2), by using the theoretical relation between $C\,(D)/C_o$ and D/L_o; see, for example, the graph marked C/C_o of Plate I of the same reference.

For the simplest case, $\alpha_o = 0$, the breaker height H_b and the depth of breaking D_b may, according to Sverdrup and Munk [SCRIPPS INSTITUTION OF OCEANOGRAPHY, 1945; SVERDRUP and MUNK 1946a], be computed by using two semi-theoretical graphs, one giving H_b/H_o as a function of H_o/L_o, and one giving D_b/H_b as a function of H_o/L_o. The first one is essentially identical with the curve 0 of Figure 1 of the present paper; it is a revision of Figure 14 of SCRIPPS INSTITUTION OF OCEANOGRAPHY [1945]. For sufficiently low initial steepness this function, according to the solitary wave theory is

$$H_b/H_o = 0.30\,(H_o/L_o)^{-1/3} \quad \ldots\ldots\ldots\ldots \quad (3)$$

Use of the second semi-theoretical graph may for most cases be replaced by simply putting $D_b/H_b = 1.28$; but, according to the Stokes theory, for waves with very large initial steepness this ratio is somewhat higher, up to 1.72 for waves with initial steepness 0.14. The curve used for computing the graphs of this paper is a revision of Figure 13 of the reference last cited.

If however, $\alpha_o \neq 0$, things are more complicated. The formula to be used is

$$H_b/H_o = (H_o'/H_o)\,(H_b/H_o') = R_b\,(H_b/H_o') \quad \ldots\ldots\ldots\ldots \quad (4)$$

where, according to (1) and (2), R_b is a function of α_o and D_b/L, and where H_b/H_o' is the same function of H_o'/L_o as, in the previous case of no refraction, H_b/H_o was of H_o/L_o. (The subscript b attached to any symbol means that the value of the quantity in question at the point of breaking is meant.)

One might at first sight be tempted to compute H_o' by means of the cube root law of the solitary

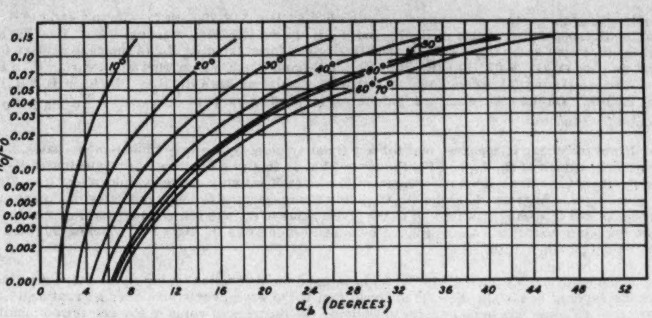

Fig. 1--Diagram for obtaining the breaker height from the initial steepness and the crest angle in deep water

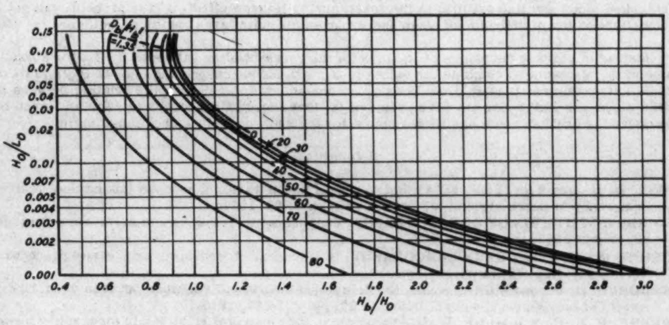

Fig. 2--Diagram for obtaining the breaker angle from the initial steepness and the crest angle in deep water

wave theory instead of from (1) [compare MUNK and TRAYLOR, 1947], but this would not be correct. As a matter of fact H_0' may be looked upon as a fictitious deep-water wave height, namely the deep-water wave height that would, without refractional effect, give the same breaker height as the actual height H_0 does with the refractional effect. In this way we discount the energy loss or gain by divergence or convergence by reducing the deep-water wave height accordingly. (It may easily be seen that the equation we obtain from (3) by substituting H_0' for H_0 is identical with Munk and Traylor's Equation (A12). The difficulty is now that we do not yet know D_b, because we do not yet know H_b. In order to overcome this difficulty we may first use an estimated value of D_b (or even a fixed value of D_b, as one does when using a so-called refraction disk) and so find a value of H_b as well as a better value of D_b, and then, if necessary, use this better value of D_b in order to find corrected values of H_b and D_b. Although for the forecasting of breaker heights the first approximation often will do, the second (or even a third) approximation is in general desirable if more accurate values of the breaker characteristics and especially of α_b, the breaker angle are wanted, for instance for the forecasting of longshore currents.

We may, however, construct graphs which make it possible to compute the breaker characteristics directly, without following a process of successive approximation or just guessing the depth of breaking in order to start the computation with. Indeed, if α_o is given, we may for any value of H_o'/L_o find the corresponding values of H_b/H_o and H_o/L_o in the following manner: First, find H_b/H_o' and D_b/H_b in the same way as H_b/H_o and D_b/H_b are found from H_o/L_o in the case of $\alpha_o = 0$. Next we have $D_b/L_o = (D_b/H_b)(H_b/H_o')(H_o'/L_o)$ from which we can find α_b, and $H_o'/H_o = R_b$ according to (2) and (1), as we have seen above. Finally we have $H_b/H_o = (H_b/H_o')(H_o'/H_o) = (H_b/H_o') R_b$. On the other hand we find the corresponding value of H_o/L_o from $H_o/L_o = (H_o'/L_o)(H_o/H_o') = (H_o'/L_o)(1/R_b)$.

From the above it appears, that for any fixed value of α_o we may find as many pairs of corresponding values of H_o/L_o and H_b/H_o as we like, so that is a diagram with coordinates H_o/L_o and H_b/H_o we can draw lines α_o = constant for a set of suitably chosen values of α_o (see Fig. 1). By entering the diagram along the (H_o/L_o)-line corresponding to the given value of the initial steepness and following this line up to the given value of α_o one can read off the value of H_b/H_o from the other coordinate axis. Figure 1 also shows the line $D_b/H_b = 1.35$. Below this line we may safely put $D_b = 1.3H_b$.

In computing the values of H_b/H_o we have used the theoretical value of the refraction factor R at the depth of breaking. Now, it is known that if the wave crests turn over a great angle, say more than 45°, one has to be careful in applying the theoretical value of the refraction factor, since, in general, the wave energy will not strictly follow the orthogonals, but may leak across these from zones of higher energy toward zones of lower energy. It is plausible, however, that in the case of straight, parallel bottom contours, where the wave energy is everywhere evenly distributed along any line parallel to the coast, such a leakage effect, if it is present, will not have much effect on the distribution of wave energy, even for quite large angles of incidence.

Instead of plotting the values of H_b/H_o with the corresponding values of H_o/L_o for a fixed value of α_o, we can also combine the values of α_b with the corresponding ones of H_o/L_o in a α_b - (H_o/L_o) diagram and so again draw lines α_o = constant, which now give the relation between the initial steepness and α_b for any given value of α_o (see Fig. 2). It appears that for any given initial steepness the breaker angle has its maximum for $\alpha_o = 60°$ or a little above that angle.

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THE GREATEST DEPTH IN THE OCEANS

H. H. Hess and M. W. Buell, Jr.

Abstract--The information concerning the deepest soundings in the oceans is reviewed. It is concluded that the previously accepted values for the Emden and Ramapo Deeps are somewhat in error and that the Cape Johnson Deep in the Mindanao Trench is probably the deepest valid sounding thus far obtained. The Cape Johnson sounding is 5740 fm after correction for the speed of sound in sea water according to the British Admiralty tables of 1939.

The greatest reported depths in the oceans are located in the Mindanao Trench off the Gulf of Leyte, Philippine Islands (see Fig. 1). The deepest sounding, 5900 fm, has been credited to the German cruiser Emden. An analysis of the Emden's observations, taking into consideration the resurvey of the area by the Dutch ship Willebrord Snellius and the traverses through the same area by the U. S. S. Cape Johnson, indicates that the generally accepted values for this depth are too large. Soundings taken by the Cape Johnson, some 45 mi to the north of the Emden Deep, indicate a depth of 5740 fm (corrected according to MATTHEWS [1939]. This depth is considered to be the greatest that thus far has reliably been discovered at sea.

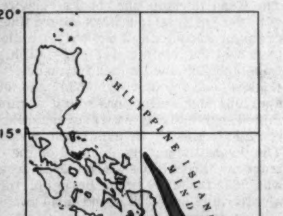

Fig. 1--Sketch map showing location of Mindanao Trench; the deepest soundings are found in the portion of the trench near 10° N latitude

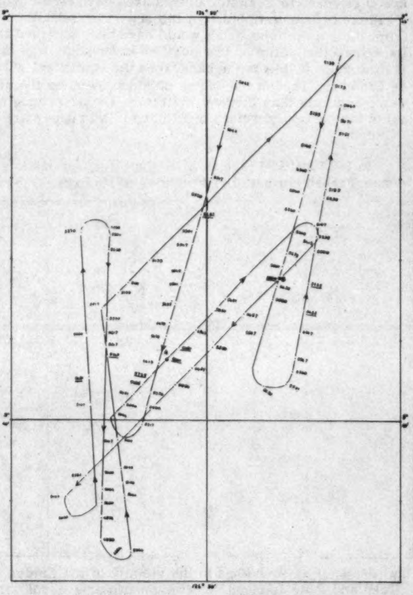

Fig. 2--Emden soundings as corrected by German Admiralty; A and B are the deepest soundings; underlined values are considered to be spurious reflections from sides of trench; note sounding of 5413 fm 300 yd from A obtained by the Emden about 15 min prior to sounding at A; the above soundings have been converted from meters to fathoms by the writers

The Emden 5900-fm soundings [MAURER, 1927 and 1928] were obtained on April 29, 1927, at 9° 40.8′N, 126° 49.6′E, and 9° 41.8′N, 126° 50.9′E (see Fig. 2). These soundings are a few miles south of Planet Deep, 5352 fm, a wire sounding made in 1912 (9° 56′N, 126° 50′E). The Emden was using an audible frequency echo sounder (non-directional). It is quite evident from a study of the Emden's sounding traverses that many of the soundings are spurious values - reflections from points far to one side and not directly below the ship. This occurrence is common in non-directional echo sounding observations over rugged topography. Though the phenomenon is well-known, it will be discussed at some length below in connection with the evaluation of the Emden's data.

The area was reinvestigated by the Snellius expedition in 1930 [VAN RIEL, 1934; and VAN HUYSTEE, 1944]. Audible frequency echo sounders were also used by the Snellius, but spurious values were for the most part avoided by careful work of the scientific staff, and slope corrections were made wherever possible. The Snellius resurvey of the Emden Deep found a maximum depth of 5555fm. One Snellius sounding six fathoms deeper than this was discarded as probably spurious. Whether the latter value be accepted or not is of small moment since the difference between the two values is negligible considering the total error involved in such soundings.

The sounding of 5555 fm (or rather, its equivalent in meters) is the corrected value as given by the Snellius Report. Some doubt exists in the writers' minds as to the exact correction for the speed of sound in seawater in this area used by the Snellius staff. This is a result of the death of F. Pinke before completion of his report on soundings. It is known that the values vary but slightly from the corrections which would have been obtained from MATTHEWS [1939]. It is probable that the correction value on this particular sounding was that derived from the observations at Snellius Station 262. If this is the case, then the equivalent value obtained by using Matthews' tables would be 5550 fm. Though the values obtained by using the data for Snellius Station 262 might be slightly more accurate than those of Matthews, the difference between the two is negligible and therefore all other corrected values mentioned in this paper are taken from Matthews as a matter of convenience.

In 1944 and 1945 the U. S. S. Cape Johnson made several sounding traverses across the Mindanao Trench almost at right angles to its axis. One of these passed less than a mile south of the charted position of the Emden Deep. The Cape Johnson line checks very well with the Snellius soundings where lines cross or soundings of one ship are close to that of the other (see Fig. 3). The Cape Johnson used a high frequency (18 kc) echo sounder, type NMC, so that spurious side reflections would be minimized, particularly on traverses almost at right angles to the axis of the trench. The deepest sounding obtained on the traverse through the Emden Deep area was 5532 fm (5230 fm uncorrected, for a calibrated speed of sound in seawater of 4800 ft/sec).

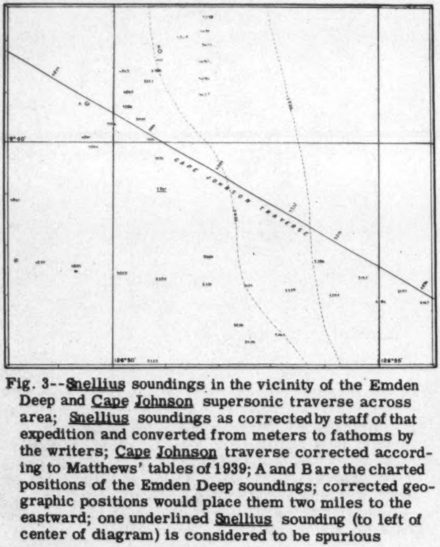

Fig. 3--Snellius soundings in the vicinity of the Emden Deep and Cape Johnson supersonic traverse across area; Snellius soundings as corrected by staff of that expedition and converted from meters to fathoms by the writers; Cape Johnson traverse corrected according to Matthews' tables of 1939; A and B are the charted positions of the Emden Deep soundings; corrected geographic positions would place them two miles to the eastward; one underlined Snellius sounding (to left of center of diagram) is considered to be spurious

In Figure 2 points labeled A and B show the positions of the Emden's deepest soundings. The same points are shown in Figure 3. If the geographical positions of the Cape Johnson and Snellius soundings are accepted (which seems warranted since they agree with each other and the Cape Johnson had the advantage of radar fixes within a few hours of the soundings), then the Emden traverses should be moved as a group two miles to the eastward to bring them into adjustment with the other soundings.

We may now turn to an evaluation of the Emden data and the general problems presented by non-directional soundings

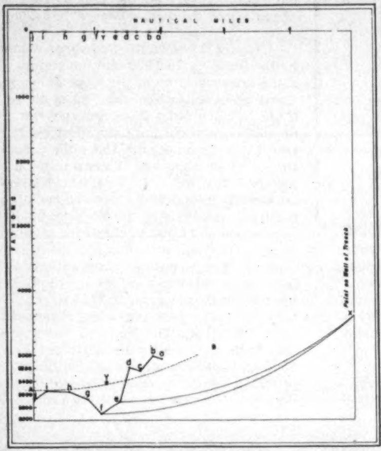

Fig. 4--Emden sounding traverse through point A of Figure 2; A equals sounding f in this figure; spurious soundings e and f probably come from wall of trench about four miles to the westward and at a depth of 4500 fm (point X of the figure); it would have been almost impossible to obtain soundings from e and f inasmuch as a non-directional echo sounder would have given echoes from points a, b, c, h, and i before a signal would be received from e or f; line A-B indicates writers' estimate of probable depth along line of profile; Y indicates sounding obtained on cross traverse about 15 min earlier than e and f; these soundings are corrected values

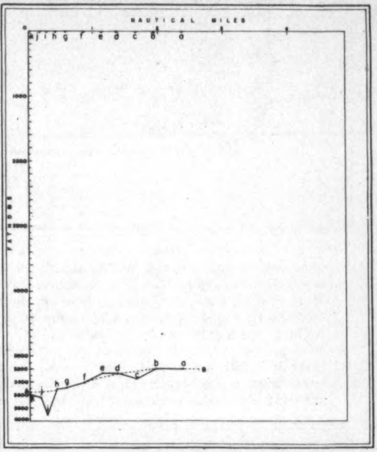

Fig. 5--Emden sounding traverse through point B Figure 2; line A-B indicates writers' estimate of true depth along line of profile; echoes from j and i would not be possible with non-directional echo sounder inasmuch as prior echoes would be received from e, f, g, h, and k; echo received at i probably from wall of trench four to five miles away and 4000 to 4800 fm deep; these soundings are corrected values

in a deep trench. In crossing such a trench good return signals are commonly obtained from the steep side slopes. These slopes are probably hard and rough. If the traverse is run at right angles to the axis of the deep, the bottom profile may be corrected for slope soundings in a fairly satisfactory manner by drawing arcs from the ship's successive positions with radii equal to the indicated depths at these positions. Tangents drawn to this series of arcs give a reasonably accurate picture of the bottom topography and the true depths. Unfortunately, the Emden traverses run nearly parallel to the axis of the trench.

The central area of the trench commonly has a gently undulating to fairly level surface and in some cases has a very flat surface such as MURRAY [1945] indicates for the Aleutian Trench. The central level area as a rule gives rather weak echoes. In many cases a half dozen or more signals have to be sent out to get one return. This level area along the axis of the trench probably represents the depth to which the trench has been filled by fine oceanic sediments and is probably a soft bottom. Such a situation is ideal for obtaining occasional spurious echoes on a non-directional echo sounder. With failure to receive a return echo from the soft bottom beneath the ship, a more distant point on the hard side slopes of the trench may, under favorable circumstances, give an echo. Many of the Emden's very deep soundings are evidently of this character as can be seen from the plot of the profiles of the traverses, Figures 4 and 5. These figures show the Emden's sounding lines, converted to fathoms, as reported by the German Admiralty in 1928. Corrections of the speed of sound in these waters are those applied by the Germans in their report. These corrections are slightly smaller than those which would have been obtained from MATTHEWS [1939], but the differences are negligible so far as present purposes are concerned. A glance at Figures 3 and 4 shows soundings several hundred fathoms shallower than the maximum values of 5900 fm at points all around these maximum points A and B in Fig. 3). The distances to these surrounding soundings are only a few hundred to 1000 yd. If the bottom configuration actually was as shown by the recorded soundings, the shallower points surrounding the deepest sounding would

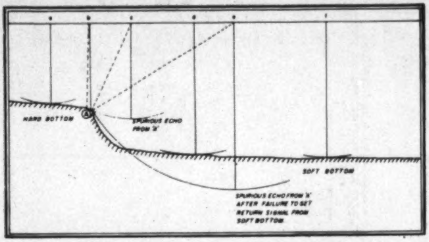
HARD BOTTOM
A
SPURIOUS ECHO FROM 'A'
SOFT BOTTOM
SPURIOUS ECHO FROM 'A' AFTER FAILURE TO GET RETURN SIGNAL FROM SOFT BOTTOM

Fig. 6--Geometrical relations which result in spurious echoes with a non-directional sounder either deeper or shallower than true depth; soundings from a, b, d, and f are equal to true depth or very close to it; sounding at c is a spurious echo from A and gives a value which is too shallow; sounding at e, over soft bottom, failed to get return from floor of trench but picked up later echo from hard ledge at A giving a value which is too deep; at d and f an echo was obtained from the soft bottom

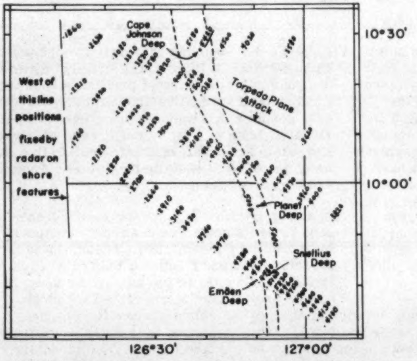

Fig. 7--*Cape Johnson* supersonic traverses across Mindanao Trench, uncorrected and for an assumed speed of sound in sea water of 4800 ft/sec; locations of Planet, Snellius and Emden Deeps also shown

have given return echoes considerably ahead of the deep point (see Fig. 6).

Figure 3 shows the soundings taken by the *Snellius* in 1930 and the supersonic traverse line of the *Cape Johnson*. Continuous soundings were taken by the *Cape Johnson* along her traverse, but depths were recorded only about every two miles, or at points where the direction of slope changed. Thus it may be assumed that the depths vary regularly between the recorded values shown along the line. Analysis of the *Snellius* soundings shows only one value which appears to be a spurious echo (underlined in the figure). The results of observations by both ships indicate a maximum depth for the trench in this area of 5555 fm corrected, the deepest soundings obtained by the *Snellius*. The *Emden*'s results alos indicate a maximum depth between 5500 and 5600 fm after elimination of probable spurious values and moving the positions approximately two miles eastward.

The four supersonic traverses made by the *Cape Johnson* are shown in Figure 7. The southernmost traverse has already been discussed since it is the one which crosses through the Emden Deep area. The maximum depths found on each of these traverses in sequence from south to north are 5230, 5150, 5180, and 5420 fm uncorrected at 4800 ft/sec. The corrected values according to British Admiralty tables of 1939 are as follows: 5532, 5442, 5475, and 5740 fm. The third traverse from the bottom of the figure was interrupted by a plane attack so that it is possible that a deeper maximum value might have been obtained had the traverse extended a few miles farther to the east. The area of the trench deeper than 5000 fm uncorrected is indicated by dashed contour lines.

The Cape Johnson Deep of 5740 fm corrected was obtained on July 14, 1945, on a course 095° true from the Gulf of Leyte and at an approximate speed of 13.5 knots. A good position fix was obtained by visual bearings on shore points and slightly later by radar at $20^{h}00^{m}$ local time. The sounding was taken approximately at midnight. Loran and sun lines determined the ship's position on the following morning. The error in position is not more than 3.5 mi. The calculated position of the sounding is 10° 27'N, 126° 36'E. The sounding was taken by R. K. Pritchard, Quartermaster, first class, USNR, an experienced and reliable operator of the echo-sounder. Pritchard had been taking soundings every day that the ship was at sea for more than a year. The average error in reading the echo-sounder is plus or minus one scale division or 20 fm. A number of soundings were taken in rapid succession and averaged to obtain the values below 5000 fm. Another source of error is associated with possible deviation in frequency from the standard value of 60 cycles of the current feeding the echo sounder motor. This was checked by reading a frequency meter. If the value departed from 60 cycles a proportional correction was applied. No correction was necessary at the time the Cape Johnson Deep sounding was obtained. An estimated current fluctuation of 0.2 cycles during the reading would involve a depth error of ±20 fm. It is believed that the sounding of 5740 fm corrected is within ±50 fm of the true

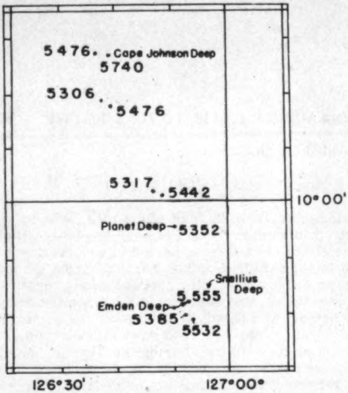

Fig. 8--Two deepest soundings of each traverse of the Cape Johnson corrected according to the Matthews' tables; Emden and Snellius Deeps and Planet wire sounding also shown

depth, taking into consideration the operator error, deviation in frequency of the input current, and error in the correction tables.

Next to the Mindanao Trench, the greatest depths so far discovered lie in the southern portion of the Japan Trench near 30° 30'N, 142° 30'E. These soundings were obtained from three old lines of the U. S. S. Ramapo, using an audible frequency echo-sounder. The deepest sounding, the Ramapo Deep, is given as 5502 fm uncorrected. A corrected value of 5771 fm appears on several H. O. charts. This sounding is unquestionably spurious and came from the nearby trench wall, as in the similar case of the Emden Deep. On the same traverse line the Ramapo has another sounding of 4284 fm only three miles 013° from the 5502 fm sounding. The distance from the point on the surface over the 5502 fm sounding to the point on the trench wall three miles away and 4284 fm down would be almost 5502 fm. The deepest valid soundings appear to be near the center of the trench rather than immediately off the steep slope, as in the case of the 5502-fm sounding. A complete reanalysis of the Ramapo traverses suggests a maximum depth of about 5360 fm uncorrected. The analysis is difficult, however, as the positions of the traverses are subject to a much larger error than was the case in the Emden Deep area, and the deep soundings themselves were apparently made by stop-watch timing which involves a possible error of about ±80 fm. The traverse on which most of the spurious values were obtained was on a course nearly parallel to the trench axis. Rolling of the ship in such a case apparently increases the chances of receiving echoes from either of the trench walls rather than from points immediately below the ship. The Ramapo's traverse, nearly at right angles to the trench axis, gives a maximum depth of 5360 fm uncorrected which appears to be a good sounding (5673 fm corrected). If this value is accepted for the Ramapo Deep it leaves that deep in the same relative position as before, namely, the second deepest sounding in the oceans.

Acknowledgments--The writers wish to thank various members of the staff of the U. S. Navy Hydrographic Office for advice and criticism during the compilation of these data, and to acknowledge assistance from funds supplied by an Office of Naval Research oontract with Princeton University (N6onr-27008) under which various oceanic areas are being investigated.

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ACTIVITY OF PARICUTIN VOLCANO FROM AUGUST 1, 1948 TO JUNE 30, 1949

Carl Fries, Jr. and Celedonio Gutiérrez

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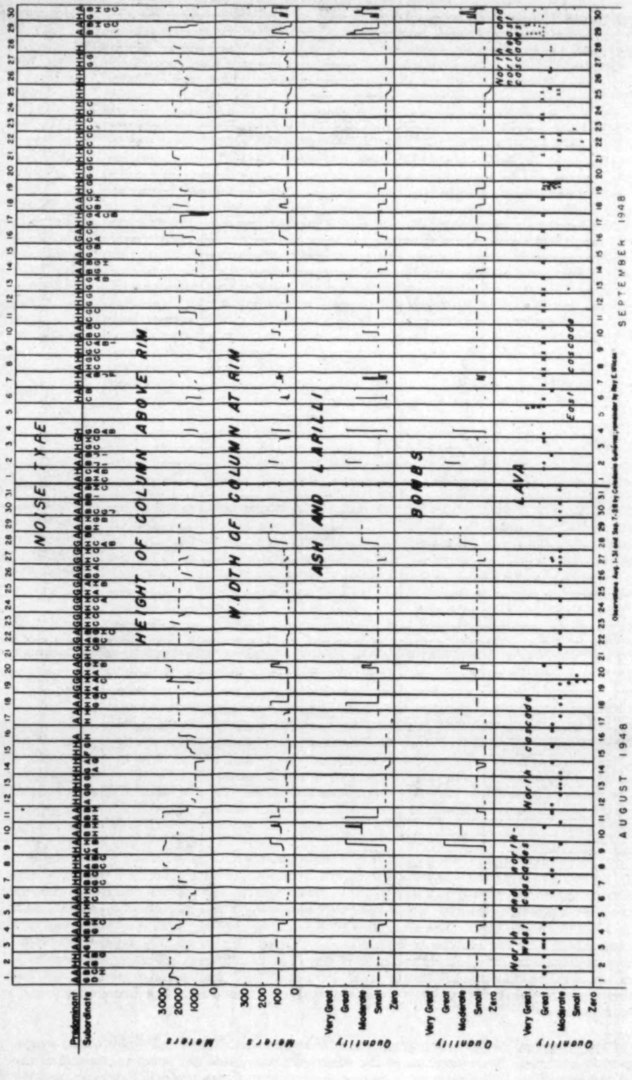
The present brief summary of activity at Parícutin Volcano, State of Michoacán, Mexico, follows the pattern of the last report on this volcano published in these Transactions late in 1948 [WILCOX and GUTIERREZ, 1948]. Anticipating the end of his two-year period of observation at Parícutin in December 1948, Wilcox instructed the junior author, a native Tarascan from the town of San Juan Parangaricutiro, now destroyed, in the processes of keeping detailed diagrammatic records of eruptive activity, of making weather observations, and of outlining the monthly advance of the new lava. On Wilcox's departure the senior author, who had closely followed the series of studies at Parícutin and had frequently visited the Volcano from June 1943 onward, took up the task of watching over Parícutin and preparing reports on its activity. During the first six months of 1949 he spent 11 days at the Volcano on the following dates: January 25-28, March 29-April 1, and May 18-20. On these visits he checked the elevations of the chief points on the rim of the cone, using an alidade at known stations, and in addition followed the growth of the lava-vent mound, variations in the form of the interior of the crater, and any other features that showed change. Further, he discussed with the junior author, the Volcano's activity during the periods between visits and assembled the observational data for formal presentation.

The diagrams of eruptive characteristics (see Fig. 1 to 6) and the maps of areas covered by new lava (see Fig. 7 and 8) are similar to those heretofore published by Wilcox, except that barometric pressures and tide-producing forces are not included, although they are kept on file, and place-name spellings are changed to conform to those being used in three reports on different aspects of Parícutin that are soon to be published in bulletin form by the U. S. Geological Survey. An explanatory discussion of the diagrams and a description of the lettered noise types appeared in an earlier report on Parícutin [WILCOX, 1947].

Activity of the cone--During the period covered by this report, Parícutin passed its sixth birthday (February 20), continuing to perform on much the same scale as in the previous two years. On the whole, the quantity of ash erupted continued to decrease, as judged from the recorded ash fall (see Table 1), and the new lava from early 1948 onward is still nearly bare of ash, but eruption of vapors and bombs continued with roughly the same intensity as the year before. The eruptive column rose at times to heights of more than 3000 m above the rim of the crater; periods of complete quiescence were few and lasted generally no more than a few minutes before eruptive activity resumed. The most heavily charged pyroclastic eruptions were accompanied by flashes of lightning within the column, as in former years, and occurred on one to five days of each month of the period covered. Eruptions came in large part from a single vent in the center of the crater, although two vents were seen when the cone was ascended on May 19, 1949 (see Fig. 9). This condition contrasts with the almost constant presence, in former years, of two or more vents in the bottom of the crater.

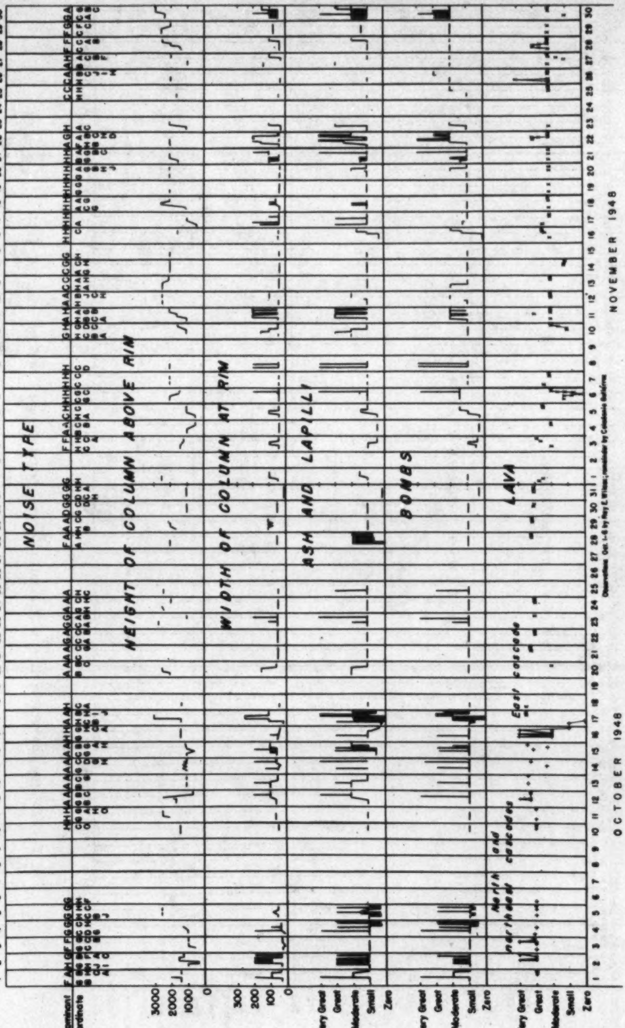
In the period January 4-9, 1949, a series of tremendous explosions occurred that could be heard as far as 200 km from the Volcano, and Earth tremors were frequent in a radius of several kilometers from the cone. On April 2 and 5, 1949, other intense explosions could be heard as far as Pátzcuaro, 140 km away. Similar explosions that shook the Earth occurred on May 13 and on June 11, 13, 20, and 28-30, 1949. The explosion of June 13 apparently tore out the center of the crater, for blocks, one to two meters in diameter, of reddish-brown slightly welded rubble were hurled several hundred meters beyond the base of the cone in the west quadrant, and smaller blocks were found more than one kilometer from the cone. Such explosions compare with the strongest eruptions of the first year of Parícutin's activity, though now they come singly and at widely spaced intervals.

Form of the cone and crater--Changes in the outward form of the cone were slight during the 11-month period covered. The elevation of the east peak was about the same at the end of the period as at the beginning. The west peak was built up nearly three meters, however, and the lowest



Observations Aug. 1-31 and Sep. 7-28 by Celedonio Gutiérrez; remainder by Ray E. Wilcox

Fig. 1--Eruptive characteristics of Parícutin Volcano from August 1 to September 30, 1948



Observations: Oct. 1-6 by Ray E. Wilcox; remainder by Celedonio Gutiérrez

Fig. 2--Eruptive characteristics of Parícutin Volcano from October 1 to November 30, 1948

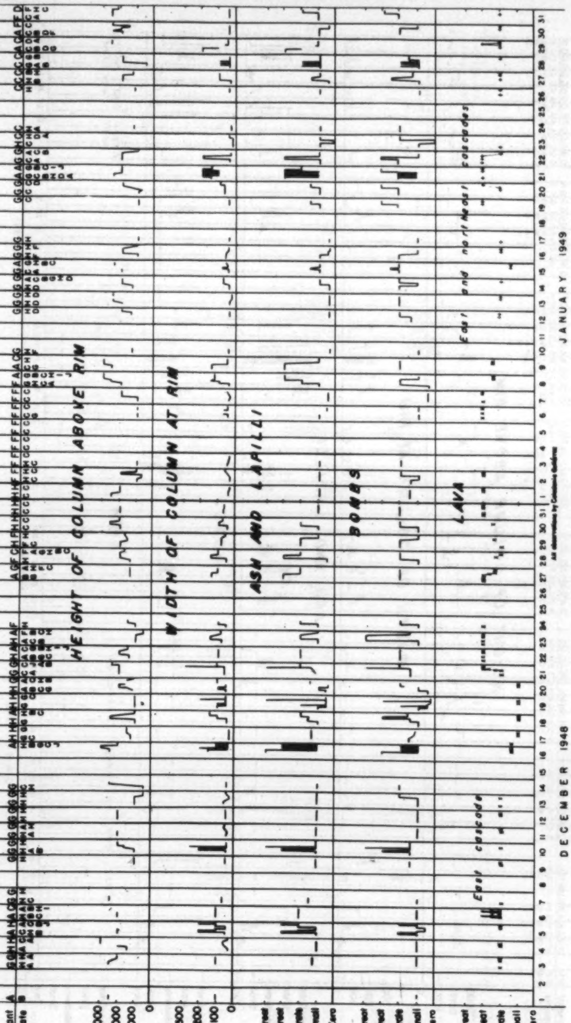


Fig. 3--Eruptive characteristics of Parícutin Volcano from December 1, 1948 to January 31, 1949

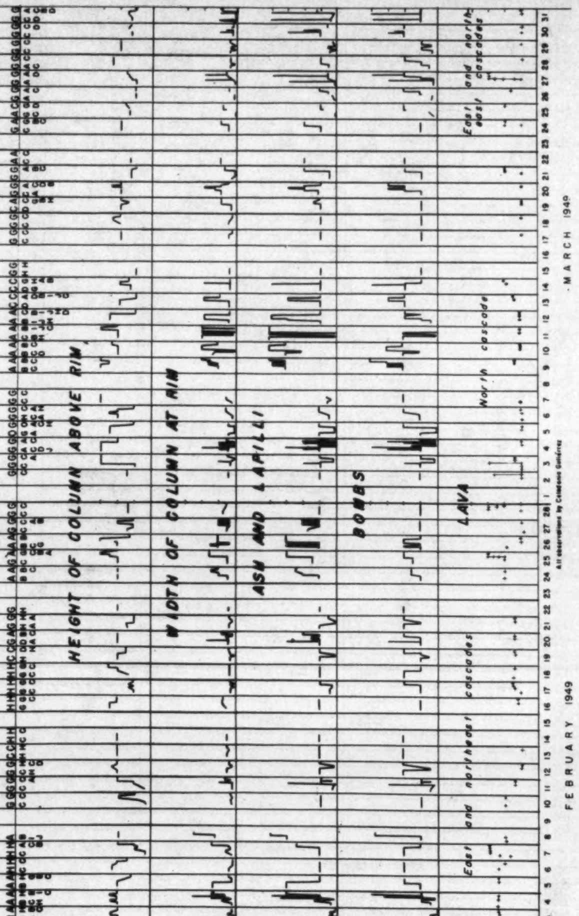


Fig. 4--Eruptive characteristics of Parícutin Volcano from February 1 to March 31, 1949

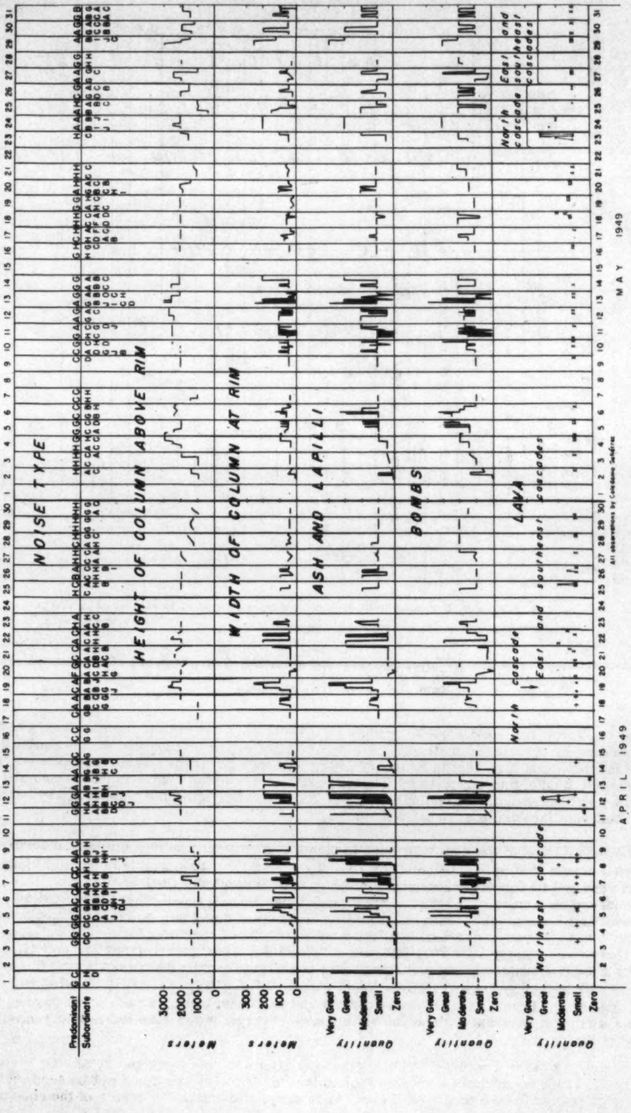


Fig. 5--Eruptive characteristics of Paricutin Volcano from April 1 to May 31, 1949

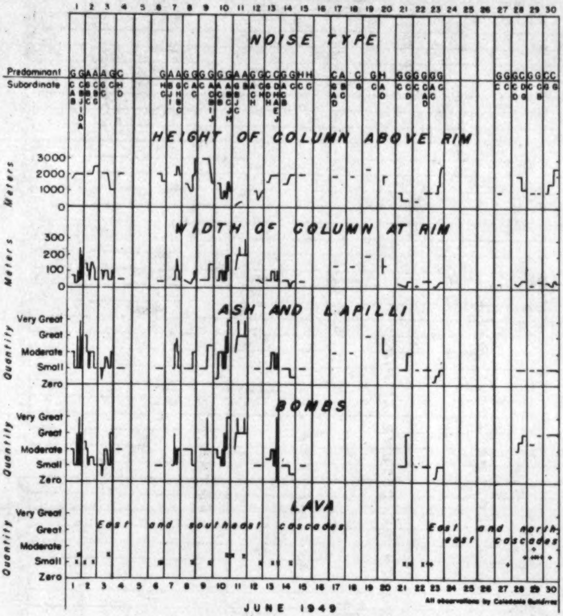
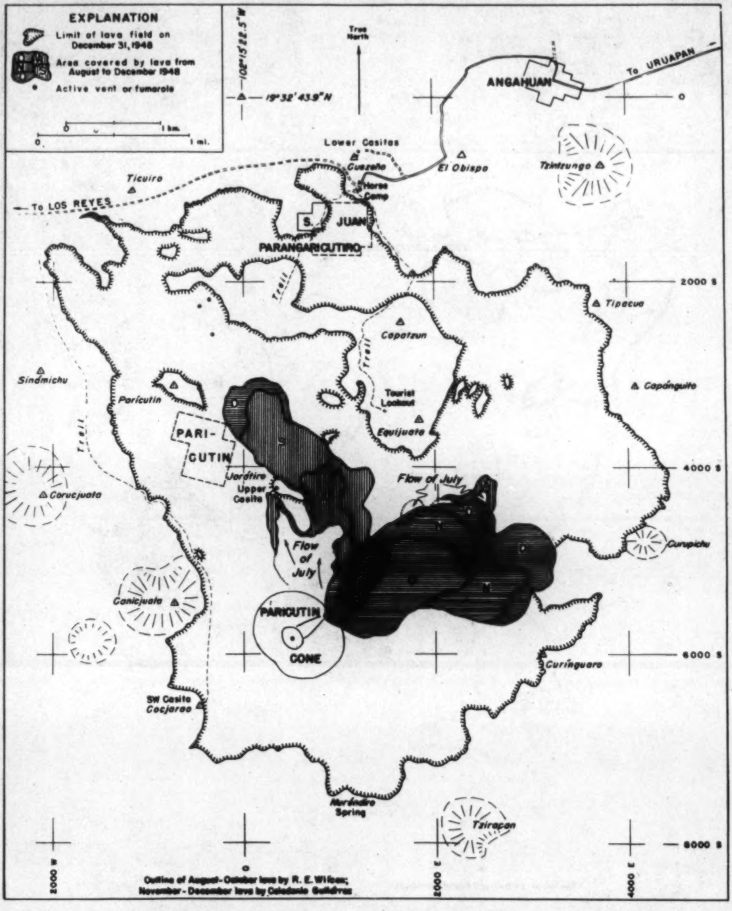


Fig. 6--Eruptive characteristics of Parícutin Volcano from June 1 to June 30, 1949

point of the northeast rim rose about 23 meters (see Table 2). This rise in the northeast saddle reflected healing by pyroclastic material from the crater after the slumps that accompanied the opening of the northeast lava vent in February 1948. Although no direct measurements were made of the southwest saddle, this appeared by inspection on ascents to the crater to have risen about as much as the northeast saddle, reflecting healing of that side also, after the slumps of 1947 and 1948 when the southwest lava vents were active.

The northeast flank of the cone continued to slump intermittently during the period covered, possibly as a result of simple erosion of the roof of the lava tube, or possibly by partial plugging of the lava vent and its forceful reopening by lava pushing through the flank of the cone. Minor slumps occurred on August 12, 1948, and again on October 14, 1948, and others appeared on November 28, 1948, and February 16, 1949, but from then until the end of June, no movement of consequence was noted. The deep depression at the southwest base of the cone, formed by collapse of the lava-vent area in April 1948 [WILCOX and GUTIERREZ, 1948], was gradually being filled with pyroclastic material erupted from the crater. The bomb-eroded rills on the northwest flank of the cone, mentioned in the last report [WILCOX and GUTIERREZ, 1948], were filled in also and have nearly disappeared. Lava flowing to the northwest and southeast buried a part of the base of the cone in the northeast quadrant, so that the visible area covered by the cone was slightly smaller by the end of the period.

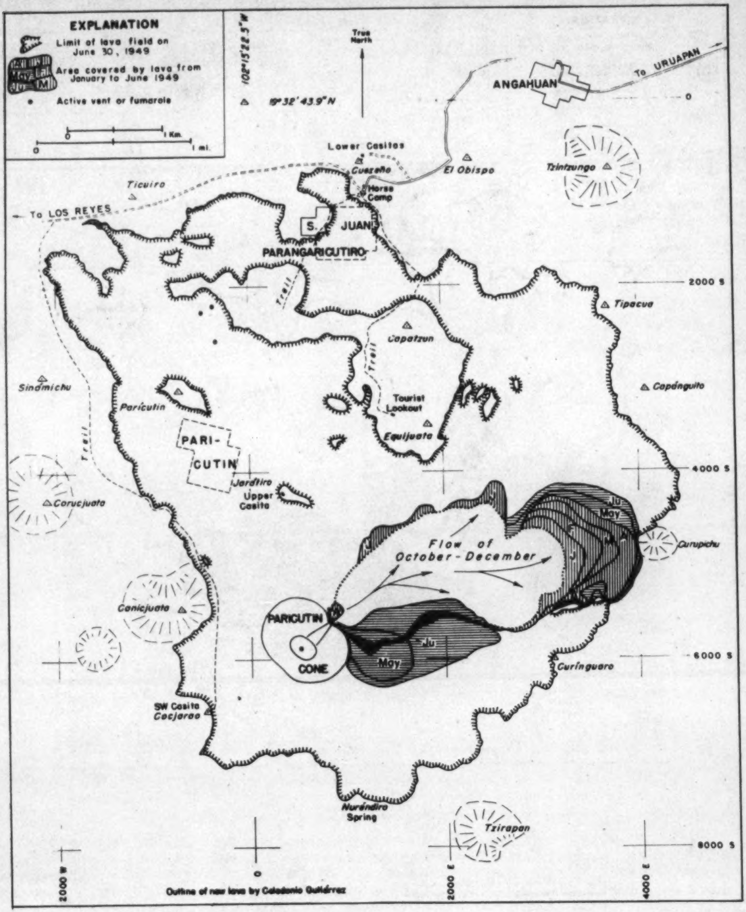
The crater, as usual, continued to show great variations in both form and depth. On the ascent on January 27, 1949, the eruptions were so violent that all of the crater could not be seen, but the central part appeared to be a single bowl about 40 m deep, situated in the center of the cone and


EXPLANATION
Limit of lava field on December 31, 1948
Area covered by lava from August to December 1948
Active vent or fumarole
0
1 km.
0
1 mi.
102°15'22.5"W
19°32'43.9"N
True North
ANGAHUAN
To URUAPAN
Lower Casitas
Guezeño
Horse Camp
El Obispo
Tzintzungo
Ticuiro
To LOS REYES
S. JUAN
PARANGARICUTIRO
Trail
Tipacua
Capatzun
Sindmichu
Parícutin
PARI-CUTIN
Tourist Lookout
Equijuata
Capánguito
Jaratiro
Upper Casita
Flow of July
Flow of July
Corucjuata
Curupichu
Canicjuata
PARICUTIN CONE
Curínguaro
SW Casita
Cocjarao
Huráncho Spring
Tzirapan
0
2000 S
4000 S
6000 S
8000 S
2000 W
0
2000 E
4000 E

Outline of August-October lava by R. E. Wilcox;
November-December lava by Celedonio Gutiérrez

Fig. 7--Areas covered by Parícutin lava flows from August 1 to December 31, 1948

surrounded by a broad terrace a few meters lower than the outer rim. There appeared to be a single wide throat. On March 30, 1949, the terrace was broader and the central bowl was shallower, probably not more than 25 m deep. The single throat consisted of a nearly vertical hole in the bottom of the bowl, with a diameter of about five meters at the top and about three meters at the point where it dropped out of sight. On the ascent of May 19, 1949, the terraces were still narrower and at nearly the same height as the outer rim of the cone (see Fig. 9). The central part of the crater consisted of a double bowl alined in a northeasterly direction and divided by a narrow, very low ridge. The bottom of the double bowl was about 20 m below the northeast saddle


EXPLANATION
Limit of lava field on June 30, 1949
Area covered by lava from January to June 1949
Active vent or fumarole
0
1 Km.
0
1 mi.
102°15'22.5"W
19°32'43.9"N
True North
ANGAHUAN
To URUAPAN
Lower Casitas
Cuezeño
El Obispo
Tzintzungo
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Ticuiro
To LOS REYES
S. JUAN
PARANGARICUTIRO
Trail
2000 S
Tipacua
Capatzun
Sinámichu
Capánguito
Tourist Lookout
Equijuata
Parícutin
PARI-CUTIN
Jaratiro
Upper Casita
4000 S
Corucjuata
Ju
May
Curupichu
Flow of October-December
Conicjuata
PARICUTIN
CONE
Moy
Curínguaro
6000 S
SW Casita
Cocjarao
Nuréndiro Spring
Tzirapan
8000 S
2000 W
0
2000 E
4000 E
Outline of new lava by Celedonio Gutiérrez

Fig. 8--Areas covered by Parícutin lava flows from January 1 to June 30, 1949

and showed two small throats from one to three meters in diameter, and several smaller openings. Eruptions from the two main throats were not of the same character; the southwest throat erupted ash and bombs and the northeast throat erupted only vapors. However, the two vents reacted nearly alike to changes in eruptive intensity. The southwest throat was by far the stronger of the two, and the walls around it were caving actively (Figure 9). On the ascent of July 6, 1949, made soon enough after the end of the period covered by this report to indicate the form of the crater on June 30, 1949, the entire central part of the crater was occupied by a conical mound and the lowest part of the interior of the crater was no more than five or six meters below the saddles (see Fig. 10). The

Table 1--Monthly summaries of weather observations, rainfall, and ashfall near Parícutin Volcano

Month	Cuezeño station							Average direction of upper winds	Equijuata station	
	Temperature		Rainfall		Ashfall		Min. humidity		Rainfall (total)	Ashfall (total)
	Max.	Min.	Total	Max.	Total	Max.				
1948	°C	°C	mm	mm	mm	mm	pct		mm	mm
August	23.9	8.3	454.4	49.0	0.0	0.0	52	. . . .	502.5	0.5
September	23.7	7.7	157.7	51.8	0.5	0.2	47	. . . .	179.7	1.3
October	25.0	6.1	145.5	38.0	0.1	0.1	34	ESE	89.0	3.0
November	24.7	6.5	128.9	57.6	3.3	1.5	40	SSW	132.0	7.7
December	24.2	4.4	13.7	11.0	0.4	0.3	17	ESE	42.0	2.0
1949										
January	23.9	2.5	0.4	0.4	0.6	0.3	26	E and SW	2.0	1.5
February	26.4	2.8	3.8	2.8	0.6	0.3	20	SSE	16.0	4.0
March	29.5	5.6	16.5	16.5	3.6	1.0	15	SSW	0.0	4.0
April	31.4	3.9	0.0	0.0	2.1	0.8	10	SW	0.0	2.0
May	31.4	7.5	92.7	28.2	tr.	tr.	16	SW and NE	115.5	1.0
June	26.4	7.5	210.9	81.3	tr.	tr.	50	ENE	382.8	0.2

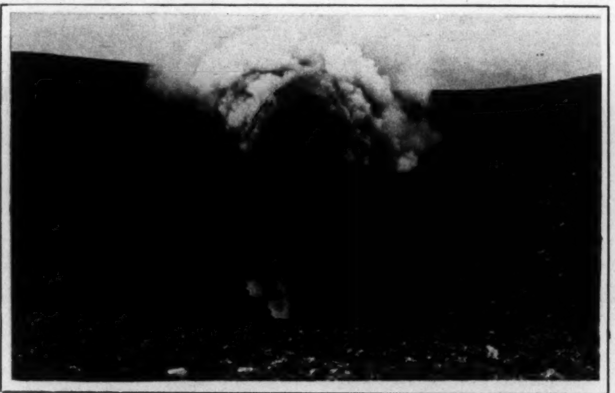

Fig. 9--Interior of Paricutin crater from north rim on May 19, 1949; note bowl-like depression about 20 meters deep, with double vent in bottom; sides are slumping (all photos by C. Fries, Jr.)

single, nearly vertical throat was about six meters across. A possibly similar process occurred in 1947 with the building up of an inner cone, the crater rim diameter of which, however, was about two-thirds that of the main outer rim. By October, 1947, the elevation of the inner rim surpassed that of the outer rim, and shortly thereafter the flanks of the inner cone coalesced with those of the main cone.

Activity of the lava vents--During the period covered by this report, lava continued to issue without interruption from the vent that formed at the northeast base of the cone on February 7, 1948 [WILCOX and GUTIERREZ, 1948], while the vent area at the southwest base of the cone remained inactive. At the beginning of August this northeast vent occupied the top of a low mound from which lava flowed in two cascades, toward the north and northwest. By August 12, 1948, the northwest cascade had ceased, but lava continued to flow toward the north until September 5, 1948, when it too ceased and a new east-flowing cascade formed. During the rest of the period, repeated surges flooded the vent area and caused changes in the direction of the cascade, but always within

Table 2--Elevation[a] above sea level of points on Parícutin Volcano

Date	West peak	Lowest point of northeast rim	East peak	Lava-vent mound
1948	m	m	m	m
July 31	2770.9	2739.5	2772.6	2621.8
Sep. 4	2770.8	2741.0	2771.7	2624.8
Oct. 1	2772.2	2741.0	2768.6	2630.5
Nov. 19	2772.3	2741.1	2771.8	2635.0
1949				
Jan. 31	2772.7	2755.4	2769.1	2635.4
Apr. 1	2772.2	2757.9	2770.1	2639.2
May 20	2772.2	2757.9	2771.7	2640.7

[a] Determined by alidade from Cuezeño station; estimated accuracy of ±1.5 meters

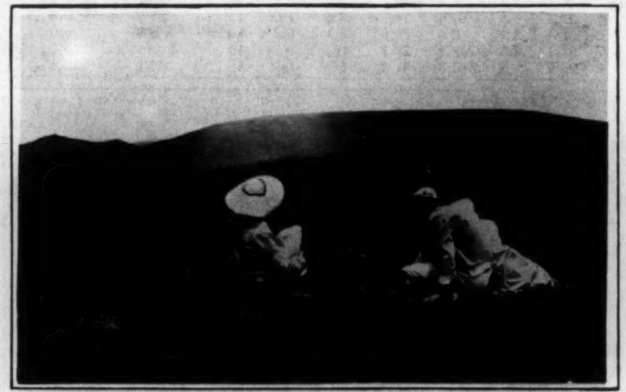

Fig. 10--Interior of Paricutin crater from east inner rim on July 6, 1949; brief lull in activity shows cone filling center of crater to height of low points on main cone rim.

the northeast quadrant (see Fig. 7-8). These floodings gradually built up the vent mound from an elevation of 2622 m above sea level at the end of July, 1948 to about 2643 m at the end of June, 1949 (see Table 2 and Fig. 11 and 12), at which time the vent was only 119 m below the northeast saddle of the cone and about 30 m below the old vent area at the southwest base of the cone. Any appreciable future rise in the elevation of the northeast vent, or plugging up of this vent by lava freezing, may possibly force the reopening of the southwest vents.

Eruptions of gases and some ash and bombs occurred from time to time at the lava vent almost every month during the period covered. Some of these eruptions were so heavily charged with ash, bombs, and vapors that they appeared like miniature atomic-bomb blasts, particularly as they always came in single puffs. These eruptions were much more frequent than in the earlier life of this vent. Other explosions at the lava vent were not accompanied by ash eruptions and resulted only in the hurling out of a scattering of pasty blobs of lava, or in puffs of blue vapors. On April 25, 1949, the only lava fountain observed during this period spurted to a height of several meters for about five minutes. At other times sudden surges caused the lava to spill over the top of the vent mound.

The depression at the southwest base of the cone, where lava had formerly issued, was gradually being filled and the area showed almost no activity. At one place, however, gases rushed out

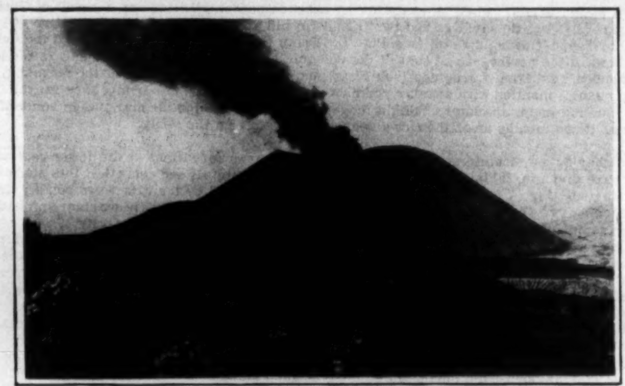

Fig. 11--Parícutin Volcano from lower casita on January 27, 1949; note lava vent mound at left base of main cone.

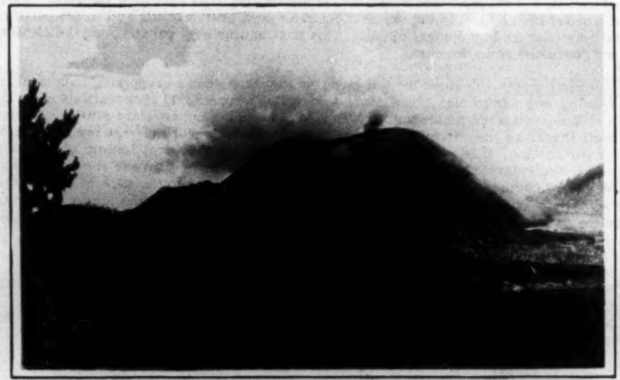

Fig. 12--Parícutin Volcano from lower casita on July 5, 1949; note how lava vent mound at left base of cone has increased in width and height, and that right crater rim is higher.

of loose rubble at temperatures ranging probably as high as 500°C, for they ignited in a few seconds any combustible material placed in their path. Fumarolic activity in this area was moderately intense during the whole period, and some cracks appeared in the ash cover on the lava late in the spring. This indicates that some movement was probably occurring there, although it was not great enough to be measured by alidade from located stations.

Activity of the lava flows -The rate of flow of the lava at the vent from August, 1948 to January, 1949, was generally moderate to great, but early in February, 1949, the volume of lava decreased notably and remained smaller until the end of June. The northwest-flowing lava stream continued to move at its front until late in October, 1948, although it had been cut off at its source nearly a month earlier, when an east-flowing cascade suddenly formed as a result of flooding. A small part of Cerro de Jarátiro and two islands of old ground to the north of Jarátiro were buried by the northwest-flowing stream (see Fig. 7). From October to the end of June all the new lava moved toward the northeast and east of the volcano, burying most of the peninsula of old ground that extended west from Curupichu (see Fig. 7 and 8). All the flows were of the normal type and were, as usual, mantled with angular reddish-brown rubble. At their fronts they ranged from about five to nine meters in thickness. Slight movement occurred along the margins of some of the lobes as late as three months after the flows ceased to be fed from the vent.

Weather data--Monthly summaries of daily weather observations at the lower casitas (Cuezeño station) are given in Table 1, together with data on total rainfall and ashfall at this station and at Equijuata (see Fig. 7 for locations). The 11 months covered by the data were notably drier than the preceding two years, and probably as a result they were also slightly warmer and had greater daily extremes of temperature. It is doubtful whether decrease in the activity of the Volcano had any effect on the weather, for average upper wind directions were somewhat different during the winter and spring of 1948-1949 from those in other years, and in Mexico City the weather during this period was also drier and warmer than usual. The greater rainfall at Equijuata station may reflect the higher altitude of that point and its position closer to Cerros de Tancítaro, which are the highest peaks in the State of Michoacán, rather than its closeness to Parícutin. Also, rainfall evaporated by the still hot lava may be re-precipitated and thus increase the total precipitation at Equijuata.

Dust storms during the spring months were much more intense than in other years. On many days the Volcano was not visible from the lower casitas after about 10^h in the morning because of the thick dust clouds, and on some days the Sun was not visible even though the sky was cloudless. On April 2, 1949, when the senior author was returning to Mexico City by auto, the wind was from the west and, as the sky was cloudless, the combined eruptive dust and wind-raised surface dust formed a broad brown band in the sky that could be seen over Morelia and to the north of Toluca before it was obscured by normal clouds. This dust undoubtedly passed over the Mexico City basin and continued on to the east.

Acknowledgments--Besides the aid given by Ray E. Wilcox in supplying data used in this report and reviewing the manuscript, the writers acknowledge occasional observations made by Jesús Saldaña at times when the junior author was absent from the observatory station. They also express their thanks to José Ramírez Betanzos for the tedious work of drafting the diagrams and maps for publication. It is particularly gratifying that the Geological Society of America has continued to pay the salary of the junior author and to cover a large share of the cost of maintaining the observatory cabins. Ezequiel Ordóñez also has contributed to the care of the observatory by paying the salary of Jesús Saldaña, who has been of great service to the junior author and has watched the camp while the latter has been away.

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México, D. F., Mexico

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USE OF HISTORICAL DATA IN FLOOD-FREQUENCY ANALYSIS

Manuel A. Benson

Abstract--A flood-frequency study for the Susquehanna River at Harrisburg, Pa., involved the use of both historical and recent flood data. The historical data started in 1786 and covered all flood peaks above a stage of 18.0 ft or approximately 340,000 cfs. The continuous record began in 1874. The use of hitherto available techniques for combining the historical with recent data was found to give inconsistent results. Analysis showed that in plotting points for the purpose of drawing a frequency curve, consistent results could be obtained only if all points were based on the same period of record. It was known that all the missing years of record since 1786 had annual peak floods of less than 340,000 cfs. It was assumed that the same distribution of peak floods below 340,000 cfs occurred during these years as is known to have occurred during those years in the recent period. The method of computation involved an adjusting of order numbers for all items below 340,000 cfs. The result is a consistent set of points and a well-defined frequency curve.

A study of flood frequencies for the Susquehanna River at Harrisburg, Pa., has led to a reconsideration of the method for computing plotting positions when combining historical floods with a recent period of record. The study was made on the basis of annual peak discharges. Probability charts based on the theory of maximum values [GUMBEL, 1945] were used for plotting purposes, although any other standard plotting procedure would have given like results.

Table 1 is a chronological listing of all known annual peak discharges at this location. The record is continuous from 1874 to 1947. During 1875 and 1885 the annual peak gage heights were caused by ice jams and the annual peak discharges are not known. From 1786 to 1868 the record of floods is taken from Water Resources Inventory Report, pt. 8 (1917), Water Supply Commission of Pennsylvania, based upon historical accounts. These apparently represent all floods above 18.0 ft from 1786 to 1874. The record for floods above 18.0 ft is therefore complete for the 162-year period 1786-1947.

Plotting positions for these peaks were first computed by the following method. The 22 peaks above 18.0 ft in this 162-year period were given order numbers from one to 22. Recurrence intervals were computed based on 162 years of record, using the formula $(n + 1)/M$ where n is the number of years of record and M is the order number. All known peaks below 18.0 ft are part of the 72-year record beginning with 1874 (1874, 1876-1884, 1886-1947). The highest of these, that for 1884, is the 16th highest in the 72-year period (since 15 years during the period are above 18.0 ft). All such peaks were given order numbers from 16 to 72 and their recurrence intervals computed based on 72 years of record.

In Table 1 are shown the order numbers assigned as explained, and the computed recurrence intervals. These are plotted against discharge on a frequency graph as shown in Figure 1. Consideration of the graph and factors involved shows that, although generally accepted methods have been used, the resulting array is not a consistent representation of the best frequency curve using all the data at hand. One set of points represents all peaks above 18.0 ft in a 162-year period; the other represents annual peaks below 18.0 ft, but for a 72-year period. The two when combined do not represent consistently the proportions of all peaks since the method of computing recurrence intervals does not combine them in the proper proportions.

Inspection of Figure 1 shows an evident discontinuity between those points above 340,000 cfs and those below. The recurrence intervals of items immediately above 340,000 cfs are computed as 163/M; those for items immediately below are computed as 73/M. Although the magnitudes of the items are very close, the recurrence intervals are necessarily far apart.

It can be seen also that the position of the lower group of points relative to the upper depends upon the length of the continuous period available. If only 50 years of continuous record were available instead of 72 (all those above 18.0 ft being known), the computed recurrence intervals for the first 22 items would be the same as previously; the computed intervals for the remainder of the

Table 1--Annual floods in chronological order, and computed recurrence intervals

Year	Month	Day	Gage height	Discharge	Order M	Recurrence interval
			ft	cfs		yr
1786	Oct.	5	22.0	462,000	6[a]	27.2
1833	May	15	18.0	344,000	20[a]	8.15
1846	Mar.	15	22.0	462,000	7[a]	23.3
1847	Oct.	9	18.0	344,000	21[a]	7.76
1853	Feb.	7	18.0	344,000	22[a]	7.41
1865	Mar.	18	24.6	545,000	4[a]	40.8
1868	Mar.	19	20.0	402,000	12[a]	13.6
1874	Apr.	22	11.3	177,000	66	1.11
1875	Apr.	4	18.0	. . . .	. .	. . .
1876	June	19	12.3	200,000	62	1.18
1877	Mar.	29	13.6	230,000	45	1.62
1878	Dec.	12	19.9	399,000	13[a]	12.5
1879	Mar.	13	14.1	242,000	39	1.87
1880	Feb.	15	10.9	167,000	68	1.07
1881	June	11	13.6	230,000	46	1.59
1882	Mar.	3	13.6	230,000	47	1.55
1883	June	28	12.6	207,000	60	1.22
1884	Feb.	15	17.2	325,000	16	4.56
1885	Apr.	5	13.9	. . . .	. .	. . .
1886	Jan.	6	19.0	373,000	16[a]	10.2
1887	Feb.	13	12.6	207,000	61	1.20
1888	Dec.	19	13.1	217,000	53	1.38
1889	June	2	26.8	620,000	2[a]	81.5
1890	Oct.	26	13.3	222,000	52	1.40
1891	Feb.	19	19.7	394,000	14[a]	11.6
1892	Apr.	5	15.0	265,000	32	2.28
1893	May	6	16.9	317,000	18	4.06
1894	May	22	25.4	571,000	3[a]	54.3
1895	Apr.	11	13.5	227,000	49	1.49
1896	Apr.	1	14.8	260,000	33	2.21
1897	Mar.	26	11.5	182,000	65	1.12
1898	Mar.	24	16.6	308,000	21	3.48
1899	Mar.	6	13.4	224,000	50	1.46
1900	Nov.	28	14.2	245,000	37	1.96
1901	Dec.	16	21.2	438,000	8[a]	20.4
1902	Apr.	10	14.9	224,000	51	1.43
1903	Mar.	2	16.6	268,000	30	2.43
1904	Mar.	27	15.1	230,000	48	1.52
1905	Mar.	21	15.9	290,000	23	3.17
1906	Apr.	16	11.2	160,000	70	1.04
1907	Mar.	16	13.3	212,000	55	1.33
1908	Mar.	20	16.0	287,000	25	2.92
1909	May	2	16.0	287,000	26	2.81
1910	Mar.	3	17.2	319,000	17	4.29
1911	Jan.	16	11.5	167,000	69	1.06
1912	Apr.	4	14.3	237,000	42	1.74
1913	Mar.	28	19.54	388,000	15[a]	10.9
1914	Mar.	30	18.02	359,000	19[a]	8.58
1915	Feb.	26	15.55	280,000	28	2.61
1916	Mar.	30	18.6	364,000	18[a]	9.06
1917	Oct.	31	12.93	210,000	57	1.28
1918	Mar.	16	15.8	282,000	27	2.70
1919	May	23	15.98	290,000	24	3.04
1920	Mar.	13	20.2	408,000	11[a]	14.8
1921	Nov.	30	15.45	273,000	29	2.52
1922	Mar.	9	12.40	197,000	63	1.16
1923	Mar.	6	14.78	255,000	34	2.15

(continued)

Table 1--Annual floods in chronological order, and computed recurrence intervals (concluded)

Year	Month	Day	Gage height	Discharge	Order M	Recurrence interval
			ft	cfs		yr
1924	Apr.	8	17.00	317,000	19	3.84
1925	Feb.	13	18.8	367,000	17[a]	9.59
1926	Nov.	17	17.0	317,000	20	3.65
1927	Mar.	23	12.95	210,000	58	1.26
1928	May	2	14.65	252,000	35	2.09
1929	Mar.	17	13.8	232,000	43	1.70
1930	Feb.	28	11.40	172,000	67	1.09
1931	Mar.	31	10.4	148,000	71	1.03
1932	Apr.	2	14.30	242,000	40	1.82
1933	Aug.	25	15.18	268,000	31	2.35
1934	Dec.	2	14.35	245,000	38	1.92
1935	July	11	12.0	184,000	64	1.14
1936	Mar.	19	30.33	740,000	1[a]	163.0
1937	Jan.	24	13.90	232,000	44	1.66
1938	Mar.	19	8.65	110,000	72	1.01
1939	Feb.	23	12.99	210,000	59	1.24
1940	Apr.	2	20.92	432,000	9[a]	18.1
1941	Apr.	7	14.35	242,000	41	1.78
1942	May	24	16.25	295,000	22	3.32
1943	Jan.	1	20.6	420,000	10[a]	16.3
1944	May	9	13.05	212,000	56	1.30
1945	Mar.	5	14.70	252,000	36	2.03
1946	May	29	22.95	494,000	5[a]	32.6
1947	Apr.	7	13.21	214,000	54	1.35

[a] Order in 162 years of record; other order numbers are based on 72 years of record.

points would be different, being based on a value of 50 years. The plotting of the lower group would change even though the 50-year period gave the same proportionate distribution of peaks below 18.0 ft as the 72-year period. The two groups cannot logically be joined by one curve while one is fixed and the other variable. It is necessary to obtain by some means that position of the lower set of points equivalent to a complete 162-year record.

In order to arrive at consistent results, it is necessary to obtain an array of peaks properly representative of the single long period. The known peaks, historical and recent, must be combined in the proper proportions in order to obtain such an array.

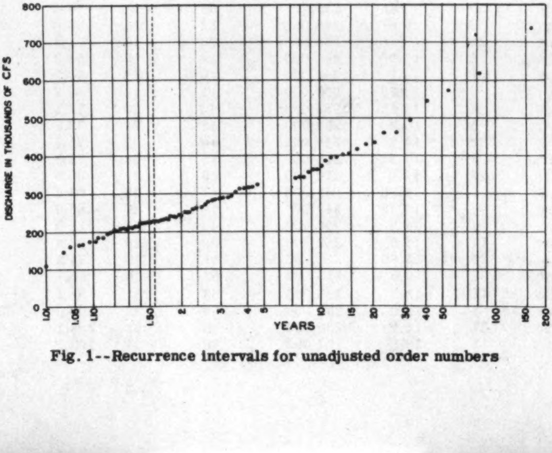

Fig. 1--Recurrence intervals for unadjusted order numbers

Table 2--Annual floods in order of magnitude, and adjusted recurrence intervals

Year	Month	Day	Gage height	Discharge	Order M	M-22	Adjusted	
							Order M_1	Recurrence interval
			ft	cfs				yr
1936	Mar.	19	30.33	740,000	1	..	1	163
1889	June	2	26.8	620,000	2	..	2	81.5
1894	May	22	25.4	571,000	3	..	3	54.3
1865	Mar.	18	24.6	545,000	4	..	4	40.8
1946	May	29	22.95	494,000	5	..	5	32.6
1786	Oct.	5	22.0	462,000	6	..	6	27.2
1846	Mar.	15	22.0	462,000	7	..	7	23.3
1901	Dec.	16	21.2	438,000	8	..	8	20.4
1940	Apr.	2	20.92	432,000	9	..	9	18.1
1943	Jan.	1	20.6	420,000	10	..	10	16.3
1920	Mar.	13	20.2	408,000	11	..	11	14.8
1868	Mar.	19	20.0	402,000	12	..	12	13.6
1878	Dec.	12	19.9	399,000	13	..	13	12.5
1891	Feb.	19	19.7	394,000	14	..	14	11.6
1913	Mar.	28	19.54	388,000	15	..	15	10.9
1886	Jan.	6	19.0	373,000	16	..	16	10.2
1925	Feb.	13	18.8	367,000	17	..	17	9.59
1916	Mar.	30	18.6	364,000	18	..	18	9.06
1914	Mar.	30	18.02	359,000	19	..	19	8.58
1833	May	15	18.0	344,000	20	..	20	8.15
1847	Oct.	9	18.0	344,000	21	..	21	7.76
1853	Feb.	7	18.0	344,000	22	..	22	7.41
1884	Feb.	15	17.2	325,000	23	1	24.46	6.66
1910	Mar.	3	17.2	319,000	24	2	26.92	6.06
1893	May	6	16.9	317,000	25	3	29.37	5.55
1924	Apr.	8	17.00	317,000	26	4	31.83	5.12
1926	Nov.	17	17.0	317,000	27	5	34.28	4.75
1898	Mar.	24	16.6	308,000	28	6	36.73	4.44
1942	May	24	16.25	295,000	29	7	39.19	4.16
1905	Mar.	21	15.9	290,000	30	8	41.64	3.91
1919	May	23	15.98	290,000	31	9	44.1	3.70
1908	Mar.	20	16.0	287,000	32	10	46.6	3.50
1909	May	2	16.0	287,000	33	11	49.0	3.32
1918	Mar.	16	15.8	282,000	34	12	51.5	3.17
1915	Feb.	26	15.55	280,000	35	13	53.9	3.02
1921	Nov.	30	15.45	273,000	36	14	56.4	2.89
1903	Mar.	2	16.6	268,000	37	15	58.8	2.77
1933	Aug.	25	15.18	268,000	38	16	61.3	2.66
1892	Apr.	5	15.0	265,000	39	17	63.8	2.56
1896	Apr.	1	14.8	260,000	40	18	66.2	2.46
1923	Mar.	6	14.78	255,000	41	19	68.7	2.37
1928	May	2	14.65	252,000	42	20	71.1	2.29
1945	Mar.	5	14.70	252,000	43	21	73.6	2.21
1900	Nov.	28	14.2	245,000	44	22	76.1	2.14
1934	Dec.	2	14.35	245,000	45	23	78.5	2.08
1879	Mar.	13	14.1	242,000	46	24	80.9	2.01
1932	Apr.	2	14.30	242,000	47	25	83.4	1.96
1941	Apr.	7	14.35	242,000	48	26	85.9	1.90
1912	Apr.	4	14.3	237,000	49	27	88.3	1.85
1929	Mar.	17	13.8	232,000	50	28	90.8	1.80
1937	Jan.	24	13.90	232,000	51	29	93.2	1.75
1877	Mar.	29	13.6	230,000	52	30	95.7	1.70
1881	June	11	13.6	230,000	53	31	98.1	1.66
1882	Mar.	3	13.6	230,000	54	32	100.6	1.62
1904	Mar.	27	15.1	230,000	55	33	103.1	1.58
1895	Apr.	11	13.5	227,000	56	34	105.5	1.54

(continued)

Table 2--Annual floods in order of magnitude, and adjusted recurrence intervals (concluded)

Year	Month	Day	Gage height	Discharge	Order M	M-22	Adjusted	
							Order M_1	Recurrence interval
			ft	cfs				yr
1899	Mar.	6	13.4	224,000	57	35	108.0	1.51
1902	Apr.	10	14.9	224,000	58	36	110.4	1.48
1890	Oct.	26	13.3	222,000	59	37	112.9	1.44
1888	Dec.	19	13.1	217,000	60	38	115.3	1.41
1947	Apr.	7	13.21	214,000	61	39	117.8	1.38
1907	Mar.	16	13.3	212,000	62	40	120.2	1.35
1944	May	9	13.05	212,000	63	41	122.7	1.33
1917	Oct.	31	12.93	210,000	64	42	125.1	1.30
1927	Mar.	23	12.95	210,000	65	43	127.6	1.28
1939	Feb.	23	12.99	210,000	66	44	130	1.25
1883	June	28	12.6	207,000	67	45	132	1.23
1887	Feb.	13	12.6	207,000	68	46	135	1.21
1876	June	19	12.3	200,000	69	47	137	1.18
1922	Mar.	9	12.40	197,000	70	48	140	1.16
1935	July	11	12.0	184,000	71	49	142	1.15
1897	Mar.	26	11.5	182,000	72	50	145	1.12
1874	Apr.	22	11.3	177,000	73	51	147	1.11
1930	Feb.	28	11.40	172,000	74	52	150	1.09
1880	Feb.	15	10.9	167,000	75	53	152	1.07
1911	Jan.	16	11.5	167,000	76	54	155	1.05
1906	Apr.	16	11.2	160,000	77	55	157	1.04
1931	Mar.	31	10.4	148,000	78	56	160	1.02
1938	Mar.	19	8.65	110,000	79	57	162	1.005

The following procedure was used. All peaks were listed in order of magnitude, starting with the highest (see Table 2). These represent 79 actual years of recorded peaks. The first 22 items of the 79 are above 18.0 ft; the remaining 57 items are below 18.0 ft. All peaks above 18.0 ft are known for the period 1786-1947, or 162 years. It is also known that for 140 years of the 162-year period the stage has been below 18.0 ft. The distribution of the flow during these 140 years is not known, but it can be assumed that the distribution is the same as is represented by the 57 years of known record which are below 18.0 ft.

Each year of record following the item 22 was taken to represent 140/57 or 2.456 years of record. The order numbers of these were adjusted by the formula $M_1 = 22 + 140/57\ (M-22)$, where

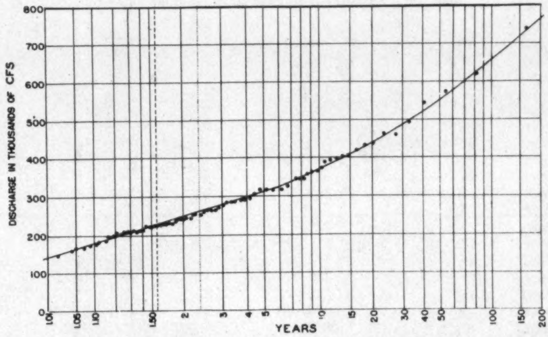

Fig. 2--Recurrence intervals for adjusted order numbers

M is the original order number and M_1 is the adjusted order number. The adjusted order number of the final item is 162. The recurrence intervals were then computed using original order numbers for items 1-22 and the adjusted numbers thereafter. The resulting array represents the distribution of frequencies for a 162-year period.

Plotting of these data (see Fig. 2) shows a continuous series of points, with the previous gap eliminated. They define very closely a curve which leaves no doubt as to its position, and appears fairly well delineated up to a frequency of at least 100 years.

Comparing Figure 2 with Figure 1, note that the positions of points above an interval of seven years are unchanged and the only appreciable change is between two and seven years. However, the plotting positions are now theoretically consistent, and although plotting positions above seven years have not been changed, the position of the curve through the higher-frequency range is more certain.

At other river stations, where there are not so many known high flood peaks, and isolated floods for varying periods of record have been used, considerable changes in the upper ends of the frequency curves are found using similar treatment.

The assumption that the 57 years of peaks below 18.0 ft represents the distribution of peaks below 18.0 ft for 140 years is seen not to be unreasonable. All annual peaks below 18.0 ft have recurrence intervals of less than seven years. It seems reasonable that 57 years of record should define the correct distribution of all events with recurrence intervals below seven years.

If a single historical flood is known and there is not much difference in magnitude between that flood and the highest in the recent period, then the method can be used with confidence. If there is a large gap in magnitude then there should be some reasonable certainty that there were few or no floods during the ungaged interval which exceeded the highest in the recent period. Fortunately this is the usual case. With a large gap and absolutely no intimation of what may have happened in between, there does not seem to be any reasonable way of properly combining the historical flood with the remainder of the data.

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SEEPAGE INTO DITCHES IN THE CASE OF A PLANE WATER TABLE AND AN IMPERVIOUS SUBSTRATUM

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Abstract--It is shown how the quantity of water seeping from a plane water table into equally spaced ditches with vertical walls may be computed by considering the ditch problem as a special case of an auger hole problem, already solved. Streamlines, distribution of hydraulic head, and the seepage rate into unit length of ditch are computed. The computed seepage rate does not account for drawdown of water table around the ditch and therefore represents a maximum value. Applications are discussed and a result compared with the work of NELSON-SKORNIAKOV [1940].

Introduction--In several papers dealing with ground-water flow, especially problems on the seepage of water into drain tubes, the author [KIRKHAM, 1947, 1949], has obtained detailed flow information analytically by solving Laplace's equation. This equation in abbreviated form is $\nabla^2\phi = 0$, and in unabbreviated form, in rectangular coordinates,

$$\partial^2 \phi/\partial x^2 + \partial^2\phi/\partial y^2 + \partial^2\phi/\partial z^2 = 0,$$

where ϕ is the hydraulic head at point (x,y,z) in the soil.

In the present paper the beauty of the mathematical attack to cover a wide range of dimensions of the flow region in question is demonstrated. It is shown how the solution of the problem of seepage into an auger hole (test well) can be utilized to solve the problems of seepage into parallel, equally spaced ditches. The auger hole solution can be used since it is exact for auger holes of all possible radii, including an auger hole of infinite radius.

Analysis--Figure 1 illustrates the problem. A series of parallel ditches is cut with vertical walls into soil of uniform permeability. The soil of thickness d rests on an impervious layer. Water stands in the ditches a depth h above the impervious layer. The ditches can penetrate below the impervious layer without changing the problem. Rainfall or other water maintains the soil completely saturated. The problem is to determine the amount of water which seeps through the soil into the ditch. Water in excess of this amount will reach the ditch as surface runoff. Since the flow is the same on each side of vertical planes parallel to, and passing midway between the ditches, only the flow in the region between one such plane and an adjacent ditch, as ABA'B', need be considered, Twice this amount of flow will be the seepage per ditch. It is further convenient to consider only the flow per unit length of ditch. Therefore ABA'B' may be considered as one face of a rectangular parallelepiped of unit thickness.

The auger hole problem from which it is proposed to derive the solution for Figure 1 is illustrated in Figure 2. The hole is of radius a and depth d. An impervious concentric cylinder of radius b forms an outer boundary for the soil. (In field cases b would be taken infinite.) Water

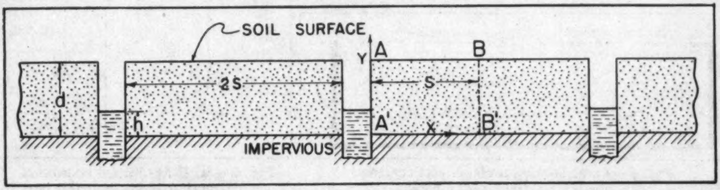

Fig. 1--Vertical section through soil resting on an impervious layer and drained by equally spaced ditch drains; section is perpendicular to ditches stands to a level h above the impervious layer.

The potential (hydraulic head) function for Figure 2 is [KIRKHAM and VAN BAVEL, 1949, esp. equations (6) and (3)]

$$\phi = d-(8d/\pi^2)\Sigma(1/n^2)(\cos n\pi h/2d)(\cos n\pi z/2d)R_0(n\pi r/2d),\ n = 1,3,5,\ldots \quad \ldots (1)$$

where r and z are coordinates of a point in the soil; and

$$R_0(n\pi r/2d) = [K_1(n\pi b/2d)I_0(n\pi r/2d) + I_1(n\pi b/2d)K_0(n\pi r/2d)]/[K_1(n\pi b/2d)I_0(n\pi a/2d) + I_1(n\pi b/2d)K_0(n\pi a/2d)] \ldots\ldots (2)$$

In (2) I_0, I_1, and K_0, K_1 denote respectively the modified Bessel functions of the first and second kinds of zero and first order. These functions have been used in these Transactions by MUSKAT [1936] and JACOB [1946] in connection with other ground-water problems.

In Figure 2 let $b-a \ll a$; then the figure assumes the form of a ring of the shape indicated in Figure 3a. If now two axial planes making a small angle with each other are passed through the ring as indicated, then lines such as AB and CD, formed by the intersection of the planes with the top of the ring, are nearly parallel. The lines AC and BD will also be nearly parallel. The figure cut out by the planes is illustrated in Figure 3b. This figure is approximately a rectangular parallelepiped, and becomes exactly a rectangular parallelepiped when b and a approach infinity, (b-a) staying finite. If the edges AC, A'C', BD, B'D' are taken equal to unity, then the parallelepiped may be identified with the parallelepiped ABA'B' of unit thickness considered in Figure 1.

It is now apparent how the flow for Figure 1 is obtained from Figure 2. One needs only evaluate (1) and (2) subject to

$$b \to \infty,\ a \to \infty,\ r \to \infty,\ a < r < b,\ (b-a) = \text{finite} \ldots\ldots (3)$$

It is shown in treatises on Bessel functions [ANONYMOUS, 1937; BOWMAN, 1938; WATSON, 1945] that for large values of argument v, the functions I_0, K_0, I_1, and K_1 have the approximate values

$$I_0(v) = I_1(v) = e^v(2\pi v)^{-1/2} \ldots\ldots (4)$$

$$K_0(v) = K_1(v) = e^{-v}(\pi/2v)^{1/2} \ldots\ldots (5)$$

Therefore, taking $v = n\pi b/2d$, $n\pi r/2d$, $n\pi a/2d$, and dropping for the moment $n\pi/2d$, substitution of the approximate values of I_0, I_1, K_0, K_1 in (2) yields for large b, a, and r, upon simplification

$$R_0(r) = (a/r)^{1/2}[e^{-(b-r)} + e^{(b-r)}]/[e^{-(b-a)} + e^{(b-a)}] \ldots\ldots (6)$$

Setting now $r-a = x$, $b-a = s$, so that $b-r = s-x$, one finds, upon passing to the limit, where $a/r = 1$

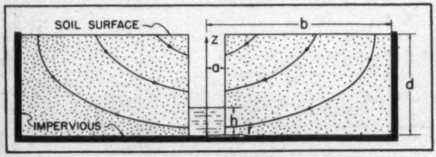

Fig. 2--Axial section through soil confined about a vertical auger hole

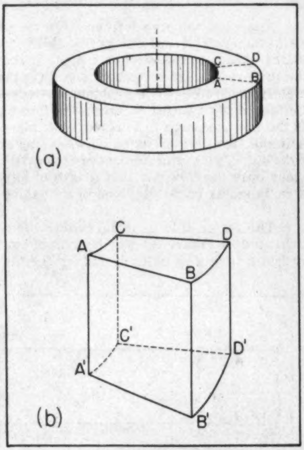

Fig. 3--(a) Ring-shaped section of soil; (b) section of soil cut from ring

$$R_0(r) = [e^{-(s-x)} + e^{(s-x)}]/[e^{-s} + e^{s}] = \cosh (s-x)/\cosh s$$

That is, restoring $n\pi/2d$,

$$R_0(n\pi r/2d) = [\cosh n\pi(s-x)/2d]/(\cosh n\pi s/2d) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (7)$$

Putting (7) in (1) and writing y for z, there results finally, the origin of coordinates now being at A′ as shown in Figure 1

$$\phi = d - (8d/\pi^2)E \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (8)$$

where

$$\begin{aligned} E = {} & (\cos \pi h/2d)(\cos \pi y/2d)[\cosh \pi(s-x)/2d)]/(\cosh \pi s/2d) \\ & +(1/3)^2(\cos 3\pi h/2d)(\cos 3\pi y/2d)[\cosh 3\pi(s-x)/2d]/(\cosh 3\pi s/2d) \\ & +(1/5)^2(\cos 5\pi h/2d)(\cos 5\pi y/2d)[\cosh 5\pi(s-x)/2d]/(\cosh 5\pi s/2d + \ldots \quad \ldots\ldots . \quad (8a) \end{aligned}$$

The quantity of water Q/2 seeping into unit length of ditch from one side may be obtained readily from either of the following expressions

$$Q/2 = P\int_0^d (\partial\phi/\partial x)dy = P\int_0^s (\partial\phi/\partial y)dx \quad \ldots\ldots\ldots\ldots\ldots \quad (9)$$

where P is the soil permeability and $\partial\phi/\partial x$ is to be evaluated at $x = 0$ and $\partial\phi/\partial y$, at $y = d$. In (9) the customary minus sign does not appear before $\partial\phi/\partial x$ and $\partial\phi/\partial y$ because the ditch is a sink, rather than a source. The symbol P is used for permeability rather than K, since the symbol K is needed later in another conventional meaning: to denote the complete elliptic integral of the first kind. From former work [KIRKHAM and VAN BAVEL, 1949] it would appear that the partial derivatives in (9) should be evaluated at $x = 0 + \epsilon$ and $y = d - \epsilon$. Careful consideration shows, however, that ϵ may be taken zero. It is found, then, using either (9), that

$$Q = (16dP/\pi^2)F \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (10)$$

where,

$$\begin{aligned} F = {} & (\cos \pi h/2d)(\tanh \pi s/2d) - (1/3^2)(\cos 3\pi h/2d)(\tanh 3\pi s/2d \\ & + (1/5^2)(\cos 5\pi h/2d)(\tanh 5\pi s/2d) - \ldots \quad \ldots\ldots\ldots\ldots . \quad (10a) \end{aligned}$$

The flux Q may also be obtained from the auger hole problem with the aid of (3), (4), and (5).

Streamlines--Since it was possible in the auger-hole case to determine only the end points of the streamlines, complete streamline expressions for the ditch case cannot be derived from the auger hole problem. However, as the ditch problem is a two-dimensional one, there are available the Cauchy-Riemann relations [cf an advanced calculus book, as SOKOLNIKOFF, 1934, esp. p. 435],

$$\partial\phi/\partial x = \partial\psi/\partial y, \quad \partial\phi/\partial y = -\partial\psi/\partial x,$$

for obtaining with the aid of equation (8), the stream function ψ. It is found that

$$\psi = (8dP/\pi^2)G \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (11)$$

where

$$\begin{aligned} G = {} & (\cos \pi h/2d)(\sin \pi y/2d)[\sinh \pi(s - x)/2d]/(\cosh \pi s/2d \\ & + (1/3^2)(\cos 3\pi h/2d)(\sin 3\pi y/2d)[\sinh 3\pi(s - x)/2d]/(\cosh 3\pi s/2d) \\ & + (1/5^2)(\cos 5\pi h/2d)(\sin 5\pi y/2d)[\sinh 5\pi(s - x)/2d]/(\cosh 5\pi s/2d) + \ldots \quad (11a) \end{aligned}$$

In plotting streamlines the factor $(8dP/\pi^2)$ may be replaced by a convenient constant. Also a constant may be added to the right hand side of (11). The constants are chosen here so as to

make $\psi = 0$ when $y = d$ and $x = 0$, and to make $\psi = 100$ when $y = d$ and $x = s$. The difference in ψ-values of any two streamlines thus represents the percentage of total flow reaching the ditch from one side.

Results--The results of the analysis are embodied in Table 1 and in Figures 4 and 5. In Table 1 multiplication of the tabulated values by $16dP/\pi^2$ gives the flow Q into a unit length of ditch. As an example; if P = 3 feet/day, d = 5 feet, h = 2 feet, s = 15 feet or more, then $Q = [16 \times 5 \times 3/(3.14)^2](0.814) = 19.8$ cubic feet/day per foot length of ditch (from both sides).

Table 1--Values of $Q\pi^2/16dP = F$ for several values of h/d and s/d; see Eq. (10) and (10a)

h/d	s/d					
	0	0.5	1	2	3	∞
0	0	0.574	0.833	0.912	0.916	0.916
0.2	0	0.564	0.811	0.887	0.890	0.890
0.4	0	0.535	0.747	0.811	0.814	0.814
0.6	0	0.469	0.625	0.671	0.674	0.674
0.8	0	0.336	0.418	0.443	0.444	0.444
1.0	0	0	0	0	0	0

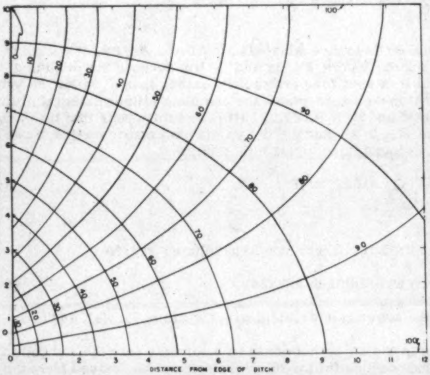

Fig. 4--Portion of flow net for a section A'ABB' of Figure 1 for the case that d = 10, h = 0, and s = ∞, the portion shown extending horizontally a distance of 12 units; units are arbitrary

Figures 4 and 5 show flow nets obtained from (8), (8a), and (11), (11a). In Figure 4 the ditch is ten units deep; the level of water in it, at or below the impervious layer. The spacing to the next ditch is infinity. The flow net covers only 12 units of horizontal distance. The equipotentials are equally spaced along the face of the ditch as they should be, none of the streamlines, except $\psi = 100$, entering the ditch wall normally.

In Figure 5 the net corresponds to the section ABA'B' of Figure 1, the ditch being half full of water and the spacing between ditches eight times the depth. The submerged portion of the wall of the ditch is at potential 0; the unsubmerged portion - the surface of seepage - is at a linearly varying potential. The streamlines enter the submerged face of the ditch at right angles, the submerged surface, at acute angles.

Applications--Mole drainage has been proposed for removing excess water from soil underlain by impermeable clay at shallow depth, as in the case of the Edina soil of southern Iowa. The mole plow is a vertical knife with a ball and chain at the bottom. The knife cuts a vertical slot through the surface soil and into the subsoil. The ball which is attached to the knife by the chain opens up a channel at the bottom of the slot as the plow progresses. This channel serves to remove water seeping down into the slot from the permeable surface soil above. In practice, mole drains are installed as close as five feet apart. Equation (10) with h = 0 gives the maximum groundwater seepage rate into mole drains for the case discussed. Figure 4 may be considered a flow net for soil at one side of a mole drain slot, the portion of the slot below the impermeable layer and the mole ball channel not being shown.

In excavating trenches in waterlogged soil, seepage water is sometimes removed by pumping. If the soil permeability is known, an upper limit for the amount of water to be pumped can, with the aid of Table 1, be computed. If the trench does not penetrate an impervious layer, and if the trench is narrow compared to its depth, an approximate upper limit for the water to be pumped can be obtained by assuming an impervious layer to exist at the depth of the trench bottom and using the results in Table 1.

If soil which rests on an impervious layer is tile-drained, the tile trenches penetrating to an impervious layer and being back-filled with gravel, then, since friction loss in the gravel is negligible, the maximum seepage rate of ground water into the tile lines can be computed with the aid of Table 1.

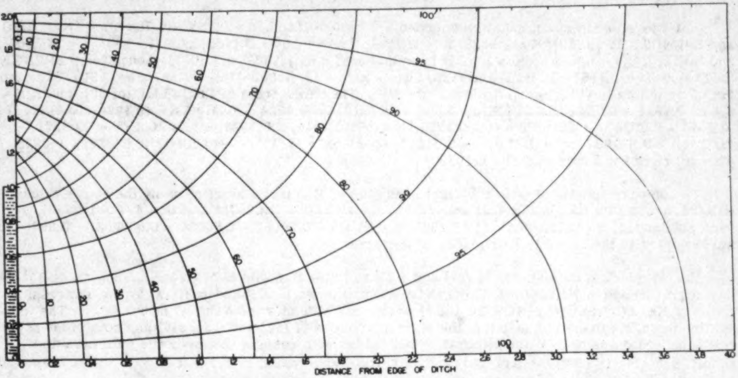

Fig. 5--Flow net for a section A'ABB' of Figure 1 for the case that
d = 2, h = 1, and s = 4; units are arbitrary

The method described in this paper is generally applicable to other problems of axial symmetry. In some cases, however, it will be easier to derive the solution of a two dimensional problem directly, than to seek for the required limiting form from the axially symmetric case.

Comparison of the theory with that of NELSON-SKORNIAKOV [1940]--The latter author has obtained some analytical results pertinent to the ditch problem here under discussion. His results do not permit, however, except by numerical integration, the calculation of flow lines, of equipotentials, or of total seepage flow. One of his results can, however, be compared with present ones as a check on the mathematical theories.

NELSON-SKORNIAKOV'S Eq. (9), in notation conforming to Figure 1, and as conventionally used in the United States, can be written in the form [if one uses the identity $\cosh^{-1} A = i \cos^{-1} A$ and divides his Eq. (9) through by P, noting that his V_y equals our $(P\partial\phi/\partial y)$ evaluated at $y = d$]

$$(\partial\phi/\partial y)_{y = d} = (2/\pi)\cos^{-1} (S/T)^{1/2} \quad \ldots\ldots\ldots\ldots (12)$$

where

$$S = (k + 1/\beta)[\text{cn } 2K(s - x)/s + \text{dn } 2K(s - x)/s] \quad \ldots\ldots\ldots (12a)$$

$$T = (1 + k)[(1/\beta)\text{cn } 2K(s - x)/s + \text{dn } 2K(s - x)/s] \quad \ldots\ldots\ldots (12b)$$

$$1/\beta = \text{dn } [2(d - h)K/s;k'] \quad \ldots\ldots\ldots (12c)$$

$$K/K' = s/2d \quad \ldots\ldots\ldots (12d)$$

and cn, dn, K, k and k' are elliptic functions and moduli defined for example by [DWIGHT [1947], and K' is the same as K but with modulus k'. From our Eq. (8) and (8a), there results

$$\begin{aligned}(\partial\phi/\partial y)_{y = d} = (4/\pi) \Big\{ &(\cos \pi h/2d)[\cosh \pi (s - x)/2d]/(\cosh \pi s/2d \\ &- (1/3)(\cos 3\pi h/2d)[\cosh 3\pi (s - x/2d]/(\cosh 3\pi s/2d) \\ &+ (1/5)(\cos 5\pi h/2d)[\cosh 5\pi (s - x)/2d]/(\cosh 5\pi s/2d - \ldots \Big\} \quad \ldots\ldots\ldots (13)\end{aligned}$$

The right hand sides of (12) and (13) should be identities. They may be checked by numerical substitution. The right hand side of (12) will first be evaluated, use being made of tables prepared by SPENCELEY [1947], (the symbol θ appearing in Spenceley being related to our k by $\sin \theta = k$.)

For convenience in computation suppose $s/2d = 0.50195$, $h/d = 0.5$, $x = 0.75s$; then from (12d) and SPENCELEY [p. 38], $K = 1.5828$, $K' = 3.1534$, $k = \sin 10° = 0.1736$, $k' = (1 - k^2)^{1/2} = \sin 80°$; and in (12c), $2(d - h)K/s = dK/s = 1.5767$, so that $1/\beta = \mathrm{dn}[1.5767, \sin 80°]$, whence from SPENCELEY [p. 318], $1/\beta = 0.4167$. In (12a) and (12b) $2K(s - x)/s = (2)(1.5828)(s - 0.75s)/s = 0.7914$. The modulus of cn and dn in (12a) and (12b) is $k = \sin 10°$. Therefore from SPENCELEY [p. 38], $\mathrm{cn}\, 2K(s - x)/s = \mathrm{cn}\, 0.7914 = 0.7044$ and $\mathrm{dn}\, [2K(s - x)/s] = \mathrm{dn}\, 0.7914 = 0.9924$. Finally, $S = (0.1736 + 0.4167)[0.7044 + 0.9924] = 1.0016$, $T = (1 + 0.1736)[(0.4167)(0.7044) + 0.9924] = 1.5091$, $S/T = 0.66371$, $(S/T)^{1/2} = 0.81468$, $\cos^{-1} (S/T)^{1/2} = 0.61862$, $(2/\pi)\cos^{-1} (S/T)^{1/2} = (0.63662)(0.61862) = 0.3938$. That is, right hand side of (12) = 0.3938.

To compare this result with the right hand side of (13) there results, using the above values of $s/2d$, h/d, and x (and noting that with odd n, $\cos n\pi h/2d = \pm 0.70711$ and that $4/\pi = 1.2732$), right hand side of (13) $= (1.2732)(0.70711)[0.42750 + 0.01049 - 0.00055 - 0.00004 + 0.000002] = 0.3938$, in agreement with the above to four places of decimals.

Having checked the validity of (12) and (13) it is now possible to evaluate $\int_0^s \cos^{-1} (S/T)^{1/2}\, dx$, an integral needed by NELSON-SKORNIAKOV. This integral multiplied by $2P/\pi$ is the expression given by NELSON-SKORNIAKOV for the flow into one side of a ditch (his Q is our Q/2). The integration is accomplished by equating the right hand sides of (12) and (13), dividing both sides of the resulting equations by $2/\pi$ and integrating both sides with respect to x over the interval 0 to s. The result is $\int_0^s \cos^{-1} (S/T)^{1/2}\, dx = (4d/\pi)[(\cos \pi h/2d)(\tanh \pi s/2d) - (1/3)^2 (\cos 3\pi h/2d)(\tanh 3\pi s/2d) + (1/5)^2 (\cos 5\pi h/2d)(\tanh 5\pi s/2d) - \dots]$

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HYDROLOGIC PROBLEMS CONCERNING THE RUNOFF IN HEADWATER REGIONS

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Abstract--In this paper are discussed some of the concepts of watershed hydrology as related to surface, subsurface, and ground-water runoff. Data from Swiss and other watershed studies illustrate that true ground-water discharge may form the major part of flood flows, with minor or no contributions by surface runoff. Results of experiments with sand troughs to determine the effect of aquifer length are used to explain apparent anomalies in the comparison of the two Emmental watersheds. It is postulated that the length of the aquifer is a factor controlling ground-water storage and sensitivity of stream flow to rainfall. In the sand troughs and the Swiss watersheds the same volume of ground-water storage did not always produce the same runoff rate. The explanation is that outflow is controlled by the rate of filling. Short rains of high intensity, even without any surface runoff, will produce crests more pointed than those of rains of longer duration and lower intensity.

Introduction--The influence of ground water on the characteristics of stream flow has repeatedly drawn the attention of hydrologists throughout the past several centuries. As early as 1686 Mariotte mentioned a problem which is not wholly solved even now: the existence of relatively stable springs. He sought the solution in the existence of subterranean reservoirs with relatively small openings, so that their yield would not vary much with heavy rains or drought. This solution may perhaps be acceptable in special cases, but not for whole basins; it is difficult to believe in the existence of many such reservoirs, one next to the other.

HAAS [1895] imputed the stability of springs in part to a regular distrubution of rain; but this does not explain springs of different characteristics in the same climate. He also thought that the spring area had to be large enough so that its draining would take more time than the period of drought. However, he did not think of sand strata of great extent, but only of long subterranean channels. This explanation is also unacceptable because it would limit stable springs to areas of large extent. But Haas took a step forward by recognizing the time needed for runoff.

LEHR [1931] sought the explanation in the thickness of layers above the ground water. According to him, in layers of great thickness there will always be water on its way to the depths, coming from rains which fell a considerable time before. Even in drought periods this water replenishes the subterranean reservoirs and so maintains the yield of springs. Thus he took into account the time required for water to reach ground water, but not that needed for runoff from the reservoir itself. Where a thick layer must be traversed in reaching ground water, this can be accepted as one of the factors stabilizing the yield of springs and rivers, reflected in lower crests and higher minima. Lehr did not mention but showed by an example (the Hirschbrunnenquelle) that in such cases there is a lapse of time between precipitation and the reaction of stream runoff. His explanation cannot, of course, be used in areas where the ground water is found close to the Earth's surface.

All three of these investigators sought an explanation in some peculiarity of the Earth, and two of them recognized the time required for water to move through the Earth. Their point of view is hydrodynamic, though still imperfect.

A totally different viewpoint is taken by Burger in Switzerland, de Haan of the Dutch East Indies, and Horton of the United States, whose conceptions do not differ much. They give attention more to whole basins than to separate springs, and see the ground water reservoirs more or less from a static aspect. A greater filling of this reservoir will mean more and longer runoff, as happens in any vessel filled with liquid. Without considering the interior pecularities of the Earth, they explain all river-flow phenomena on the basis of ground-water supplies and the quantities of water lost as surface runoff and evaporation. They reason as follows: When the topsoil is more permeable, the soil will take in more precipitation and less will run off over the surface. More water in the soil means more sustained yields in dry periods; less surface runoff means lower crests at flood time. Because surface runoff moves faster than runoff through the soil, peak-flood discharges will vary directly with surface runoff rates; and a high infiltration capacity will mean more stable stream flow.

This general rule is attributed to Horton and acknowledged by Burger and de Haan, who try to obtain greater infiltration capacities primarily with forest cover. Horton also admits the good properties of forest soil, but sees the problem more broadly and does not exclude other methods. In repeated investigations Burger proved that forest soil is more permeable than other soils of the same geologic origin; but he also found that forests evaporate more water than other vegetation. Forests will increase ground-water supplies by lessening surface runoff, but will inflict losses by increasing evaporation. The net effect depends on the ratio between these gains and losses.

It seems safe to say that this theory had been acknowledged by most hydrologists up to about 1935. It may appear logical, but is still incomplete, it takes into account only the factors regulating inflow and neglects those regulating outflow. Some facts can be explained by it, while others contradict it. A study of Figure 1 will show the difference between the theory based on these assumptions and a theory of my own, based on fewer assumptions and more empirically ascertained facts. Figure 1a conforms to statements of BURGER [1945, p. 181].

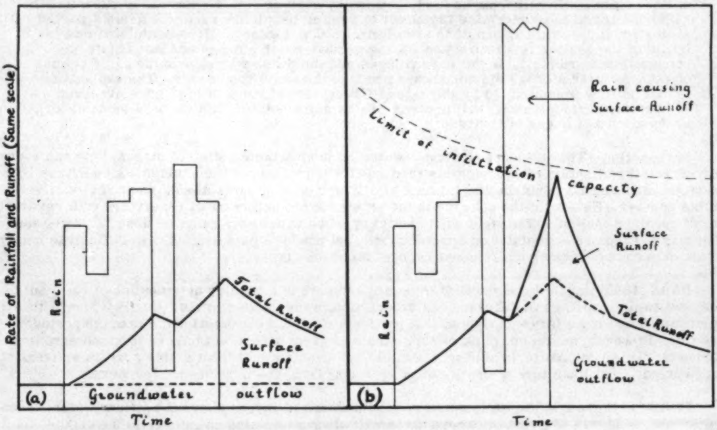

Fig. 1--Relations of rainfall and runoff with time, (a) theory of H. Burger; (b) theory of the author

The forest sponge theory was originated by those who were called "forest enthusiasts" by Horton. They thought that rains were swallowed and retained by the sponge-like mass formed by fallen leaves, which would give back the water at a slow and gradual rate in periods of drought. With some exceptions this theory is now abandoned and the imaginary sponge properties are replaced by others, experimentally found in forest soil, which still make the forest serve as a natural regulator of stream flow.

The followers of this theory are proponents for forest conservation. In 1878 Grebe showed many data indicating deterioration of the regularity of river flow after deforestation [see WOLLNY, 1879, p. 481]. While Grebe considered the connection between forest cover and an abundant yield of springs as proven, Wollny called these data very valuable, but was nevertheless unconvinced.

The theory of the forest enthusiasts may be tested by using the obvious connection between the height of ground water and the yield of springs: It will be supported if ground-water levels are higher under forests than under field cover. Ototzki and Toski in Russia, Henry in France, Pearson in British India, Stappenbeck in the Argentine, and Ijjasz in Hungary, however, found ground-water levels lower under forests than under adjacent fields, In Norway, Thurman found that deforestation raised the ground water; and in Switzerland, J. Hug observed that the yield and regularity of springs increased after deforestation [see BURGER, 1943, p. 393]. Ebermayer and Hartmann in Bavaria did not find any difference. The contrary, that ground-water levels were higher under forest cover, was never stated. It seems logical that the lower levels under forest cover must be explained by greater evaporation.

Table 1--Yields of water as related to forest cover, Java, 1925

Area	Forest cover	Total runoff, 1925	Runoff in four dry months, July-October		Maximum monthly runoff	
					Depth	Month
sq km	pct	mm	mm	per cent of total	mm	
110	100	1377	333	24.2	67	Nov.
52	98	1563	208	13.3	38	Oct.
94	22	2613	269	10.3	62	Oct.
58	16	2006	460	22.9	80	Nov.
106	12	2411	248	12.3	62	Oct.

In Java, I traced the relation of forest cover to river yields in the long drought of 1925. In three adjacent basins on the volcanic formation with 52-, 26-, and 85-per cent forest cover, the relative yields were about 20, 11, and 10. Table 1 shows similar comparisons for other basins, not adjacent but on the same formation. No connection can be seen between forest cover and the river yields during the dry period. If, however, we compare different geologic formations, both under forest cover, the influence of the soil is clearly expressed. As compared to the yields shown in Table 1 for loose volcanic formations in Java, the very compact marl soils on the same island yield nothing after two or three months of drought.

From the American investigations of Bates and Henry at Wagon Wheel Gap, Colorado, and of Hoyt and Troxell in Southern California, several American investigators concluded that forests do not conserve water because after their removal from watershed lands, the yearly runoff at Wagon Wheel Gap increased 15 per cent, and in California 29 per cent [GROSS, 1938]. The increase was not limited to periods of flood. At Wagon Wheel Gap runoff from melting snow was increased 40 per cent and summer runoff 12 per cent; winter runoff was unchanged. The summer increase led these writers to state as an "outstanding fallacy" the general belief that forests or other vegetative cover have the capacity to cause increased summer runoff by saving up the water. For loose soils I am in agreement with this opinion.

In 1935, prominent American hydrologists suggested that surface runoff is the principal source of floods in rivers during heavy rains or periods of melting snow [LEE, 1935]. Associated with this opinion is the belief that water taken into the soil will be of no importance to runoff for some time after infiltration. This belief brought BURGER [1945] to the theory expressed in Figure 1a, which has been modified by a number of American hydrologists since its inception. At Wagon Wheel Gap, for example, the maximum daily discharges were considered to be from subsurface flow, as practically all of the snow had melted several days before the peaks [GROSS, 1938]. The San Dimas watershed in California yielded over 50 per cent runoff from one very large storm, while surface runoff from plots was only two per cent; the difference was called "wet-weather seepage" [LOWDERMILK, 1938]. According to M. D. Hoover (by correspondence), this kind of runoff is also called subsurface storm runoff, quick return flow, or quick seepage - all indicating that this type of flow is considered to be different from normal ground-water outflow. HORTON [1942] called it "concealed surface runoff," not to be confused with "true ground-water outflow." He suggested that this flow occurs close to the surface through a layer of coarse material, a thick mat of grass or mulch-cover, a layer of plant roots close to the surface and under the forest litter, or even a network of Sun cracks in the soil surface. These are all possibilities; but have they been confirmed?

Let us see what can be learned from the Swiss watershed investigations, started in 1903 and still being carried on [BURGER, 1934, 1943, 1945]. Two basins, on the same geologic formation, were compared to demonstrate the influence of forest and non-forest vegetation on runoff. One basin, the Sperbelgraben, was almost entirely covered with forest; the other, the Rappengraben, was only one-third forested. In this kind of study one cannot be sure that observed differences in runoff are due to the vegetation; the method used in America at Wagon Wheel Gap and at the Coweeta Hydrologic Laboratory is much better. Apart from this weakness the data given by the investigators (Engler and then Burger) are very accurate. As shown in Figure 1a, Burger himself was not yet aware of quick seepage, concealed surface runoff, or quick reaction of the ground-water outflow. Since part of my theory has also been evolved in America, perhaps my published views on this

Table 2--Infiltration into pasture after Burger

Ten-minute period	Infiltration	
	mm/min	in/hr
1	0.43	1.02
2	0.27	0.64
3	0.25	0.59
4	0.22	0.52
5	0.19	0.45

subject [ROESSEL, 1939, 1941] will now receive recognition.

The direct reaction of ground-water outflow--Ground-water outflow as used by the writer includes all forms of gravity water moving to the stream channel through the macropores of the soil. The experiments with artificial rain [BURGER, 1943], may be used to show amounts of water taken up by the soil. He used plots one meter square, which he watered artificially with ten liters or ten mm rainfall depth in five minutes. This amount was applied five successive times with five-minute intervals between applications, so as to give a total of 50 mm of rain in 50 minutes. The intensity was two mm per minute (50 mm in 25 minutes of actual rainfall time). As Burger caught the surface runoff a half meter below the plot, we have to allow 1.5 square meters of surface area for infiltration. Rounding off his results, the surface runoff for good forest was zero, for young forest two per cent, and for pasture 60 per cent. Using the data in his Tables 9 and 10 [BURGER, 1943], we can compute the infiltration capacities for these different kinds of cover. From Table 9, which shows the greatest surface runoff, the pasture infiltration rates are shown in Table 2. The long, low-intensity storms of this region (Landregen) have average intensities below 0.10 mm/min (0.237 in/hr), with peaks mainly between 0.10 and 0.20 mm/min. So even the least pervious vegetation type, the pasture, will not produce surface runoff in this kind of storm; and therefore the hydrographs of their runoff will be composed purely of ground-water outflow.

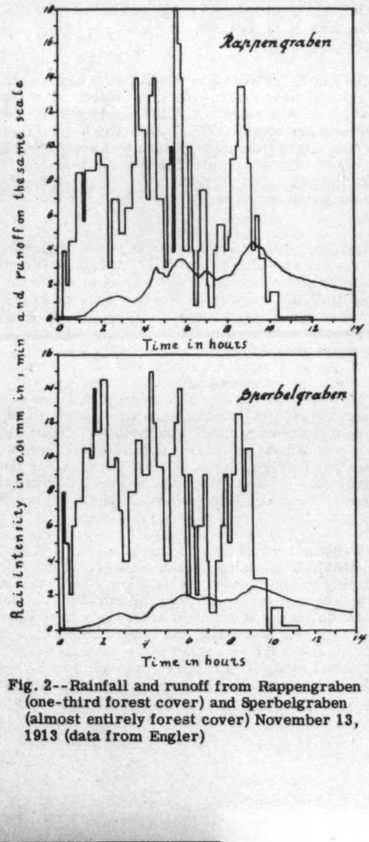

Fig. 2--Rainfall and runoff from Rappengraben (one-third forest cover) and Sperbelgraben (almost entirely forest cover) November 13, 1913 (data from Engler)

In Figure 2 are shown data for the thunderstorm of low intensity of November 13, 1913. The highest rainfall intensity is 0.18 mm/min (0.43 in/hr); and obviously the zone of field moisture (above the capillary fringe) was not yet saturated. We can see clearly that the ground-water outflow reacts directly to changes in rainfall intensity. Also, the flow of the Rappengraben is higher than that of the Sperbelgraben, which is normal for these basins; and the Rappengraben is more sensitive in its reaction to rain. Since surface runoff is not involved we have to conclude that sensitiveness to rain can be associated with ground-water outflow. This will be explained later on.

In Figure 3 are shown data for the storm of July 9, 1909, with extensive, steady rain of low intensity (Landregen), part of a rain period of several days. The antecedent rains have saturated the zone of field moisture. This graph can be compared with Figure 2, in which runoff decreases sometimes while the amount of rain still exceeds the outflow. Part of the rain is used to fill soil-moisture deficiencies and does not reach ground water. In contrast, the reaction of the flow in Figure 3 can be compared with that of a surface-storage reservoir during rain. When the rainfall exceeds the outflow, the water table will rise and the discharge rate will increase; and vice versa. As this phenomenon appears clearly in Figure 3, the conclusion is drawn that we are dealing with a ground-water reservoir and not with concealed surface runoff.

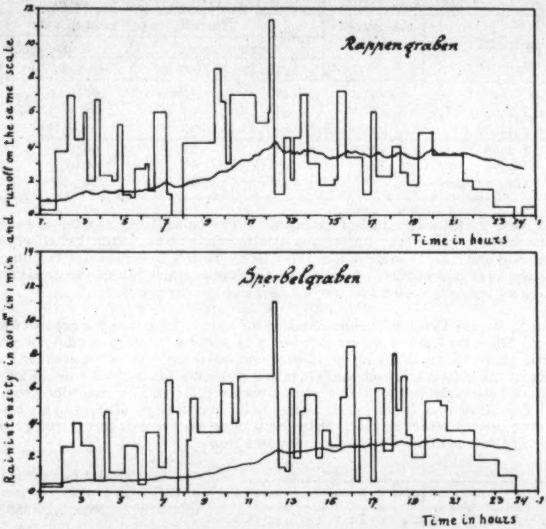

Fig. 3--Rainfall and runoff from Rappengraben and Sperbelgraben, July 9, 1909 (data from Engler)

Table 3--Watershed behavior as shown by accumulated values

Period	Watershed	Accumulated		Runoff trend
		Rain	Runoff	
Midnight to 12^h50^m	Sperbelgr.	24.4	7.722	Increasing
	Rappengr.	29.0	14.630	Increasing
12^h50^m to midnight	Sperbelgr.	24.0	18.023	Increasing
	Rappengr.	20.1	23.544	Decreasing

In this storm, 49.1 mm (1.94 inches) of rain fell on the Rappengraben, and 38.174 mm (1.51 inches) ran off, whereas the Sperbelgraben received 48.4 mm (1.91 inches) and its runoff was 25.745 mm (1.02 inches). Comparing the two watersheds for the 1909 storm (see Fig. 3), we see that the discharge crests do not come at the same time, but are about eight hours apart. This is evidently because in the Sperbelgraben the outflow is less, and in the Rappengraben more than the volume of rain as shown in Table 3. These figures accentuate the fact that we are dealing with a ground-water reservoir. In the last tabulated period, with less rain than the Sperbelgraben, the Rappengraben has more runoff. If the discharge were largely concealed surface runoff the opposite would have occurred, because the Sperbelgraben (with forest cover) has the loosest soils near the surface.

As a related fact, filling of the ground-water reservoir causes increasing outflows. Runoff late in a storm must be greater than near its beginning; and the longer the rain the greater the percentage of runoff must be. This principle is confirmed by the data in Table 4 (from Burger).

The immediate reaction of ground-water discharges to a drop in the reservoir may seem a bit strange. But it does not mean that all the ground-water flows out immediately; it is simply a reaction of discharge to changing pressure associated with a rise or fall in the ground-water table. The ideas of KING [1899] of runoff in the mountains differ from current concepts. He wrote: "How great the percentage of surface runoff is we do not know, but it is sure to be much less than would

Table 4--Relation of runoff per cents to length of storm

Storm length	Measured at end of	Runoff, as per cent of rain			
		Sperbelgraben		Rappengraben	
		1915-27	1927-42	1915-27	1927-42
	hr	pct	pct	pct	pct
Thunderstorms	12	15	17	22	26
Rains of 1 day	24	24	21	33	26
2 days	48	31	33	43	42
3 days	72	39	40	46	47
Longer rainy periods		42	50	54	64

be supposed upon a first unconsidered judgment. It can be set forth as a broad proposition that almost all the water in rivers and little lakes has its origin in the water that slowly but constantly oozes through the soil.'' It is strange that this statement of a conscientious investigator is ignored by the hydrologists of this century. It seems unquestionable that changes in ground-water elevation must have an immediate effect on the discharge at nearby points.

Where aquifers are shallow, the time needed for rain to influence the ground-water table must be very short. When the field-moisture deficiency is satisfied, a drop of rain does not need to traverse the zone above the capillary fringe to exert its influence. Its added pressure at the top will permit a drop at the bottom to reach the fringe and make the water table rise. Where the aquifers have thick layers above them, more time will be required for rain to affect the ground-water table, and this will appear as a lag in the runoff hydrograph: the peak runoff may come hours, days, weeks, and even months after the rain. Still, there is no need to call this a true ground-water flow and that from shallow aquifers a false ground-water flow.

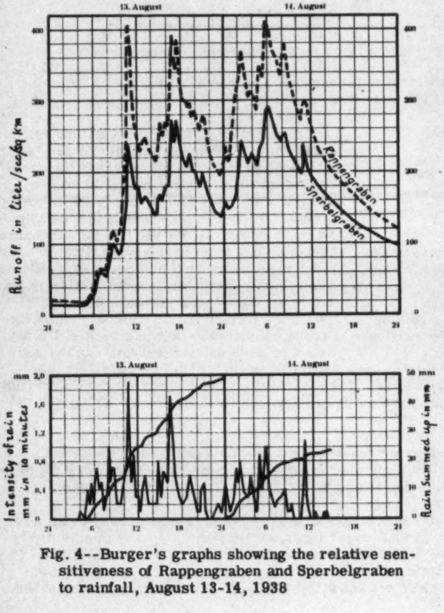

Fig. 4--Burger's graphs showing the relative sensitiveness of Rappengraben and Sperbelgraben to rainfall, August 13-14, 1938

In Figure 4 is reproduced Figure 7 of BURGER [1943]. At two points the rain intensity reaches 0.19 and 0.17 mm/min (0.45 and 0.40 in/hr); the rest of the peaks are only 0.11 mm/min or less. This and the fact that the highest runoff from the Rappengraben does not exceed 410 lit/sec sq km (37.56 cfs/sq mi) is sufficient to indicate that no surface runoff occurred. Burger writes that in this case the rain was so nearly the same in both basins that it was impossible to set it up separately. Runoff from the Rappengraben was more sensitive to rain than that from the Sperbelgraben. Burger also commented, without explanation, that the sensitiveness was greater on the second day than on the first in both basins. The data of Table 5 relate to Figure 4. This figure shows clearly the difference in sensitiveness. While Burger and other hydrologists tried to explain it as a difference in surface runoff, we know that this cannot be the cause. Perhaps the greatest ground-water discharge from the Rappengraben might be explained by a difference in evaporation; but after two to seven days, this discharge becomes less than that from the Sperbelgraben as shown by Figure 5, reproduced from BURGER [1943, Fig. 12].

This intersection of the two runoff lines always occurs; usually after the rain, but sometimes in the middle of a rainy period. The latter are exceptions;

Table 5--Rainfall and runoff reactions in a two-day rain[a]

Item	Sperbelgraben	Rappengraben	Difference
Rain in mm	71.7	72.3	0.6
Runoff in 48 hrs., mm	25.9	36.0	10.1
Runoff, per cent of rain	36	50	14
Highest peak, lit/sec/sq km	290	406	116
Runoff at start of rain, lit/sec/sq km	13	22	7

[a] To convert to English equivalents: cfs/sq mi = lit/sec/sq km × 0.0915.

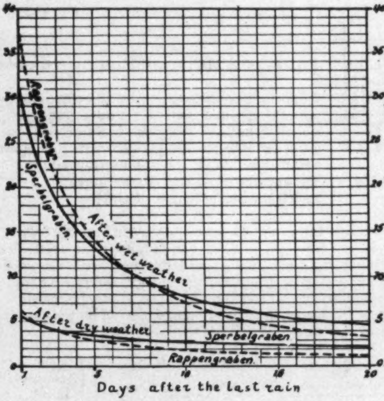

Fig. 5--Recession of runoff in the Swiss basins after wet and dry weather with about the same temperature, after BURGER [1943]

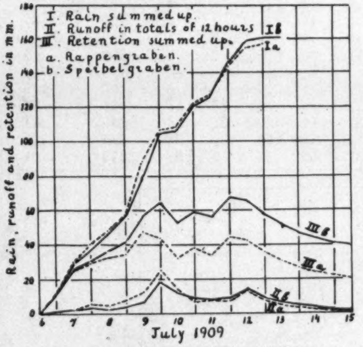

Fig. 6--Rainfall, runoff and retention in a so-called abnormal case rain period July 6-15, 1909, after data from Engler

called abnormalities by Engler, who suggested that in these cases the forest had lost its capacity for retention. Though Engler cited three such exceptions, Burger mentions none. Still, they are present in the data: The Sperbelgraben produced greater runoff in one out of 16 thunderstorms; five out of 38 one-day rains; one out of 26 two-day rains; one out of 11 three-day rains; and in no cases out of five longer rain periods.

After noting the great differences between forest and pasture in his experiments on permeability and surface runoff, it would have been very difficult for Burger to explain these exceptions by greater surface runoff from the forested watersheds. Engler, in discussing these exceptions, remarked that the water, though absorbed by the soil, came out again in the same quantity as it went in. So it is clear that he did not regard this phenomenon as being caused by surface runoff.

In one of the abnormal cases cited by Engler (July 6-12, 1909), runoff was normal at first and became abnormal during the last days of the storm, starting about noon on July 10. Rainfall and runoff and their difference (retention) are plotted in Figure 6. It is shown that the Sperbelgraben needs a greater storage of ground water than the Rappengraben to give about the same runoff.

In Figure 7 is shown the melting of snow during dry and sunny weather, as published by BURGER [1943]. As the maximum temperature is 12°C (54°F), according to CLYDE [1929] the maximum equivalent of melted snow should be 0.033 mm water per minute. As the pasture soil can absorb 0.19 mm per minute, we can be sure that only ground-water outflow can occur. The temperature in the Sperbelgraben was lower than in the Rappengraben, so it is understandable that the Sperbelgraben runoff was somewhat smaller. But the difference in sensitiveness is clearly seen; the runoff variations are much smaller in the Sperbelgraben.

Experiments with sand--Laboratory experiments were undertaken to seek an explanation for some phenomena not accounted for in the preceding discussion. The most

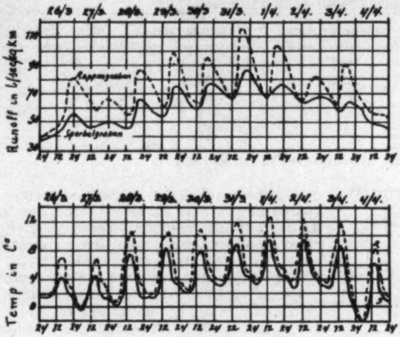

Fig. 7--Melting of snow March 26-April 4, 1934 after BURGER [1934]

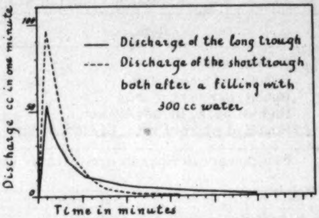

Fig. 8--Experiment with sand-filled troughs

important of these are why in comparing watersheds are there (1) a difference in sensitiveness of the ground-water outflow; (2) a crossing of the ground-water outflows after some days; and (3) a difference in the amount of ground-water storage to reach the same height of outflow.

The laboratory studies were designed to determine the effect of length of aquifer upon ground-water outflow. For this purpose sand troughs were used and the results can only be compared with basins where the aquifers have a physical resemblance to sand. We should not compare them with cases where the ground water is under pressure. In my opinion a comparison with the basins in Emmental, the Melera, Wagon Wheel Gap, Southern California, and the basins on volcanic soil in Java is permissible. It is possible that our concept of aquifer length in nature, based on the distance from ridge to valley, may be false because of the presence of underground ribulets.

Table 6--Results of periodic watering of sand troughs (eight-minute intervals)

Watering	Trough	Water added	Outflow at one minute intervals								Total outflow	Accumulated retention
			1	2	3	4	5	6	7	8		
			cc	cc	cc	cc	cc	cc	cc	cc	cc	cc
1	Short	300	101	67	41	25	16	10	6	4	270	30
2	Short	300	106	74	44	26	17	11	6	4	288	42
3	Short	300	108	76	44	28	17	11	7	4	295	47
4	Short	300	110	74	46	28	18	12	7	4	299	48
5	Short	300	109	76	46	28	18	12	8	5	302	46
1	Long	300	37	24	17	14	11	10	9	8	130	170
2	Long	300	47	32	24	22	19	18	16	15	193	277
3	Long	300	49	37	30	27	24	22	21	20	230	347
4	Long	300	57	42	35	32	28	26	24	23	267	380
5	Long	300	57	44	38	32	30	28	26	24	279	401

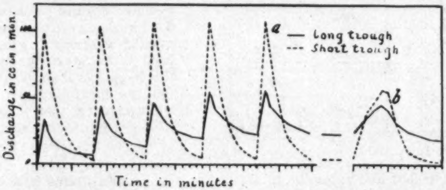

Fig. 9--Experiment with sand-filled troughs, (a) five waterings at intervals of eight minutes with 300 cc water in about ten seconds; (b) a 6th time with 300 cc water in 4.5 and 4.75 minutes

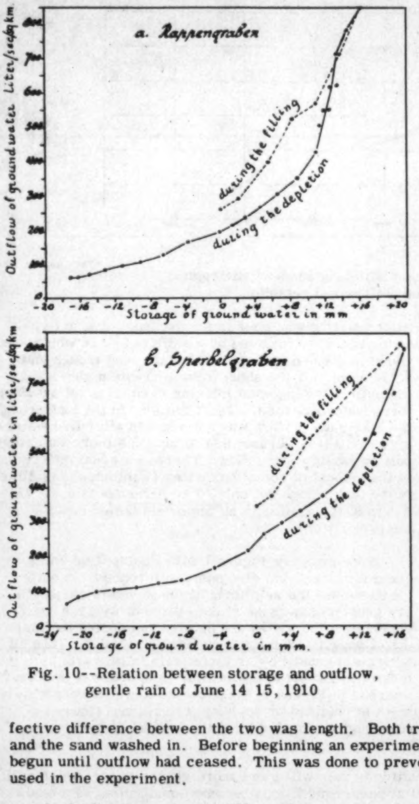

Fig. 10--Relation between storage and outflow, gentle rain of June 14 15, 1910

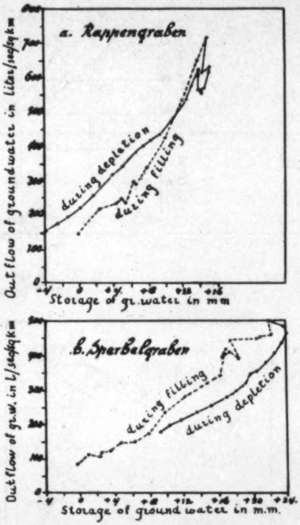

Fig. 11--Relation of storage to outflow, gentle rain of July 8, 1909

Experiments were made with two sandfilled troughs each of the same surface area and depth but differing in length and width. The measurements were 40× 20 × 20 cm and 80 × 10 × 20 cm. To eliminate the effect of the difference in width a drain of brass gauze was laid along the underpart of both troughs. The only effective difference between the two was length. Both troughs were set at an angle of 30 per cent and the sand washed in. Before beginning an experiment the trough was watered and the test not begun until outflow had ceased. This was done to prevent retention in the sand of part of the water used in the experiment.

In the first experiment, 300 cc of water, spread as equally as possible, was poured upon the sand surfaces in about ten seconds. The outflow was measured at intervals of one minute. Theoretically, one would expect that the highest runoff rate in the long trough would be only half of that in the short trough because in the long trough the distance of flow is twice that of the short trough. As shown by Figure 8, the experimental result is in accord with this theory.

Comparison of Figure 8 with Figures 4 and 5 shows that they are similar. In the beginning the short trough gives more outflow but the yields are less later on. The Rappengraben basin does the same. The ground-water outflow of the Swiss basins has the same peculiarities as the sand troughs of different lengths and can be explained in the same manner. However, according to the data of Engler the mean distance from ridge to valley is shorter in the Sperbelgraben. It is the invisible drainage of the Rappengraben which causes the reversal in their outflow patterns. In his description Engler mentions that the Rappengraben has more and larger springs and they were more constant in flow than those of the Sperbelgraben. Therefore, water flows a shorter distance to an outlet in the Rappengraben basin.

In a second experiment, the sand troughs were watered five times at eight minute intervals;

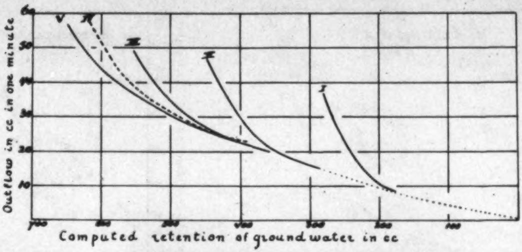

Fig. 12--Experiment with long sand-filled trough, ratio between outflow and retention

each watering was done in ten seconds. These five waterings were followed by a sixth of 300 cc which was added in a 4.5-minute period for the long trough and 4.75-minute for the short trough. Table 6 shows that the outflow during eight minutes is equal to inlfow after three waterings for the short trough. In the long trough the maximum outflow was not reached after five waterings. Still it is obvious that after some more waterings outflow would equal inflow. There is a great difference in the amount of retention. After 40 minutes it is 401 cc in the long trough but only 46 cc in the short one. The length of the aquifer is an important factor controlling the process of storage.

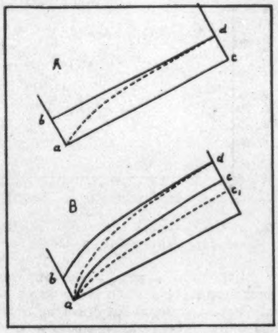

Fig. 13--Explanation of variations of outflow with storage

If we compare Figure 7 with Figure 9 we see a great similarity but also some differences. In both cases we see the amplitude of the waves of one basin are greater than those of the other but with the sand experiment the waves are pointed and during the melting of snow in nature the waves are rounded. This difference is caused by artificial rain falling with high intensity in a short space of time in the case of Figure 9 whereas in Figure 7, the snow melting proceeds slowly. A more rounded form with the sand experiment is obtained by applying water more slowly as shown by Figure 6b. The similarity is great enough to conclude that the lengths of the aquifers have caused the differences.

Generally speaking the basin less sensitive to rain will store more ground water, which will have the effect of increasing the runoff until at some time it may become equal to that of a basin more sensitive to rain, or under certain circumstances, the runoff peak may even be higher. This is an explanation of the cases described as abnormal by Engler. The length of the aquifer, by influencing the sensitiveness to rain and the minimum yield during droughts, is one of the factors which regulate the regimen of rivers.

Relationship of ground-water storage to outflow--For two storms on the Swiss watersheds, the storage of ground water was computed beginning with the time when it was estimated that deficiency of field moisture was satisfied. Figures 10 and 11 were obtained by plotting this computed storage against outflow. In three cases we see that a certain storage of ground water can correspond to different amounts of runoff, depending on whether the reservoir is filling or emptying. In the fourth case the form of the curve obtained is different perhaps because there was still a deficiency of field moisture. For the second experiment with the long sand trough the storage was computed for each minute.

Figure 12 shows that the curves I to V (corresponding to the separate runs) begin with an abrupt fall but afterwards change to a line common to all five curves. This may be considered a

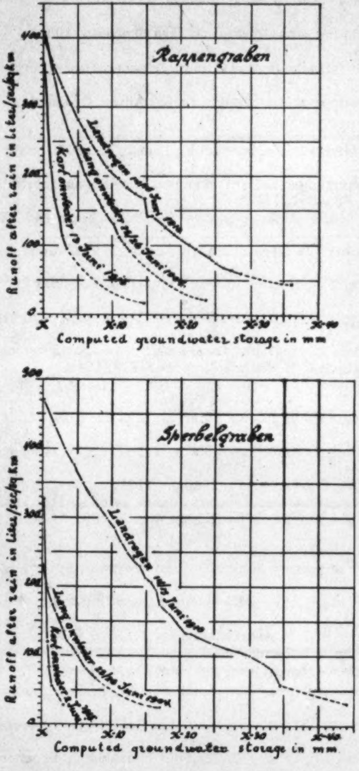

Fig. 14--Relation between ground-water storage and runoff after rains of various duration (Landregen = long rain of low intensity; lang Onweder = long thunderstorm; kort Onweder = short thunderstorm; after data from Engler)

base line and logically considered must be directed to the origin. To explain the fact that the same amount of storage produces different quantities of runoff, Figure 13 was drawn.

Imagine in Figure 13 A that filling of the reservoir was so rapid that there was no lateral flow; then the water would occupy the space abcd, with a phreatic line or water table bd. This is impossible and the water will form a phreatic line adapted to the conditions. That will be line ad. The water nearest the outlet will meet the least resistance and will flow the fastest. The outflow of abd will be faster than that of adc. The same reasoning can be followed for a second watering as we see in Figure 13 B. After the first fast outflow of the part abd the phreatic line ad will be reached adapted to the conditions. Then the outflow will be more regular and in accordance with the storage in the trough. The phreatic line will fall slowly and will reach the line ac, ac_1 until no water remains. The outflow during this period will follow the base line of Figure 12.

This shows that the faster watering takes place, the more rapid will be the initial outflow and the more distant will be this runoff from the base line of Figure 12. Figure 14 shows that this same phenomenon occurs in nature. In this figure, the relationship between runoff (after the end of rainfall) and ground-water storage is given for three different kinds of storms; a short thunderstorm, a storm of 27 hours, and a rain of low intensity lasting for 38 hours.

We can conclude that short rains of great intensity, even without any surface runoff, will produce crests more pointed than those of rains of longer duration and lower intensity. The problem of the direct reaction of the water which infiltrates into the Earth can be solved from investigations of the reaction of the springs in basins with shallow aquifers. If the springs show the same direct reaction as the whole runoff, it will confirm that we have to do with outflow from an underground water table and not with a concealed surface runoff.

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INTERCEPTION STORAGE OF RAINFALL BY CORN PLANTS

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Abstract--The processes resulting in interception and interception storage of rainfall by corn plants are discussed and terms defined. Values for two components of interception storage on mature corn plants have been determined under several conditions. The dominant factor affecting the value for residual interception storage on mature corn is the plant population. The amount and rate of rainfall have little effect on residual storage after sufficient rain has fallen to fill the storage pockets. Maximum residual interception storage on mature corn was found to be about 0.02 inch for a corn plant population of 15,000 plants per acre. This amount of storage is much smaller than has generally been accepted on the basis of earlier studies.

Introduction--The determination of ground rainfall, that is, the amount of rainfall that reaches the ground surface, is one of the first considerations in many hydrologic problems. Ground rainfall rates can be determined from rain gage measurements only through a knowledge of the rates at which rainfall is stored on the plants. Reported studies of the amounts of rainfall stored on plants vary greatly in the amounts indicated, though there has been general agreement that these amounts are great enough for careful consideration and further study. Much of the earlier work has involved comparisons between rainfall catch in the open and rainfall catch under vegetation.

Vegetation will intercept, or interrupt the course of, a portion of the rainfall, and that portion which has had its course interrupted has been termed interception. The process is cumulative as the rain progresses. The portion of the interception that is on the vegetation at any specific time is generally termed interception storage. During a rain, vegetation may not be able to retain all the water that falls on it, so part reaches the ground by dripping and by stem flow. The portion of interception which reaches the ground by dripping and stem flow can be regarded as temporary or transitory interception storage. The portion of interception, remaining on the plants when dripping and stem flow ceases, can be regarded as residual interception storage. A fraction of the residual storage may later reach the soil due to disturbance by wind, but for the most part it is disposed of by evaporation.

Ground rainfall for any specified period of time is therefore total rainfall less residual and transitory interception storage plus additions by dripping and stem flow. Residual interception storage cannot contribute to runoff, infiltration or any hydrologic process dependent on ground rainfall.

To avoid unnecessary repetition the term residual storage will be subsequently used in this paper in place of residual interception storage, and transitory storage will be used in place of transitory interception storage.

Our observations have led us to the following conception of the processes resulting in interception storage on corn plants. As rainfall begins, part is intercepted by the foliage. A portion of the rainfall intercepted is retained as storage on the plant, while the remainder drips and splashes to the ground. These processes continue until the pockets holding water begin to overflow, which results in flow down the stalk. Further interception causes the drip and stem flow to continue while residual storage approaches a maximum. Most of the residual storage appears to be in pockets at the junction of the sheath and blade, while residual storage on the leaves is mostly in the form of drops which constitutes a lesser part of the storage.

The amount of transitory storage should be a function of the antecedent rainfall rate as well as the plant population and stage of growth. The net effect of this transitory storage on ground rainfall rates is simply a delay in the time that it reaches the ground. We consider that most of the transitory storage on full grown plants should reach the ground within a minute or so after being intercepted. With this assumption, the delay would be unimportant in most hydrologic problems.

The amount of residual storage depends on the stage of growth and other characteristics of the plant as well as the number of plants on an area. It should, however, be almost independent of the amount and characteristics of the rainfall after sufficient rain has fallen to fill the pockets. The residual storage can, for the most part, be accounted for during the first few minutes of rainfall.

A review of the literature to obtain residual interception storage values indicates that considerable data are available on forest interception, but only a few data are available on agricultural crops. Most of the data available on agricultural crops appear to furnish information on interception rather than interception storage.

Review of literature--MUSGRAVE and NORTON [1937] measured rainfall in jars under a corn canopy and in the open. Their data are summarized in a table entitled "Canopy interception of rainfall, 1935," and furnish values for corn ranging up to 0.495 inch for a 2.1 inch rain. Computed on an individual plant basis, using a corn plant population of 10,000 plants per acre, 0.495 inch would amount to 7.4 lb of water intercepted by each corn plant. In the discussion of this table they refer to the intercepted water as being retained on the foliage.

REIMANN, VAN DOREN, and STAUFFER [1945] measured rainfall under a corn canopy and in the open to obtain percentage of rainfall penetrating a corn canopy for individual rains during two corn crop seasons. Their technique was similar to that of MUSGRAVE and NORTON [1937] except that pans were used to measure the water penetrating the canopy. When that portion of the rainfall not penetrating the corn canopy is computed from their data the values range up to 0.75 inch for a 2.44 inch rain. The amounts intercepted were considered as losses in arriving at the amount of water utilized by corn. During the 1944 crop season, interception was considered to have caused a loss to ground rainfall of two inches, thus reducing to 8.2 inches the amount of rain utilized by the corn.

Leonard Schiff (unpublished), seeking interception storage data for hydrograph analysis, conducted a comprehensive review of the literature. He compiled a series of tables for each crop, stage of growth, and amount of rainfall. His values were as high as 0.36 inch interception storage for a rain of 2.1 inches on mature corn. He considered these values to be too high and suggested the need for additional data.

PHILLIPS [1926], working at Deepwalls, South Africa, noted interception gains of several hundred per cent by comparing the rainfall as measured by a vegetation screened gage and a control gage. Interception gains were especially high during rains of a fine, misty nature and these gains were substantiated by observations. Phillips also measured interception loss by comparing results from a rain gage under the forest canopy with one in the open. Data for a two-year period indicated an interception loss of about 22 per cent, with losses relatively greater during the fine misty rains. In other words, there was appreciable interception loss below the trees but within their crowns the total catch was far greater (30-80 per cent) than at equivalent heights in fully exposed sites.

GRAH and WILSON [1944], using a precise weighing technique and artificial rain, determined several components of interception on Baccharis pilularis (an evergreen shrub) and Monterey pine. Data were obtained on interception, as a per cent of total simulated rainfall, as well as amounts of the several interception storage factors. Their data indicate that on most of the samples of Baccharis, maximum interception storage under a 0.70 in/hr spray was reached in less than 25 min and on most samples of Monterey pine in less than 15 min. They reached the following conclusions: (1) The maximum surface detention of water held on a plant at the end of a shower has been divided into three types of storage: (a) transitory storage, or the water that will drain from the plant under still air conditions; (b) conditional storage, or the additional water that can be removed by wind or forceful shaking; (c) residual storage, or the water that can be removed only by evaporation. (2) Under still air conditions, 72 per cent and 68 per cent of the maximum surface detention for Baccharis pilularis and Monterey pine, respectively, can be retained as interception; and under severe wind conditions, 35 per cent and 34 per cent can be retained. (3) Interception storage capacity varies between 0.01 and 0.04 inch depth for Monterey pine transplants and between 0.02 and 0.06 inch for Baccharis (includes b and c above and is calculated on the basis of the crown area of the plants).

Methods and procedure--Residual interception storage on corn can be measured by either of two general methods. One method is to measure all the rainfall penetrating the corn foliage and subtract this amount from rainfall measured in the open. The measurements under the corn foliage must include both drip and stem flow as well as rain penetrating the foliage without being intercepted. One difficulty is the variation in rainfall from point to point. This can be minimized, however, by sufficient replications. Another difficulty is catching the stem flow. This necessitates either construction of a catchment apparatus near the base of each corn plant or making the ground

Table 1--Description of mature corn plants selected for interception storage determinations

Lot number	Height	Color	Condition of plant	Number ears per stalk	Position of ears
	ft				
A[a]	6-8	Green	Turgid	1 and 2	Erect
B	8-9	Green	Turgid	2	Erect
C	6-9	Brown	Dry	1 and 2	Down
D	8-9	Green	Turgid	2	Erect

[a] A few vines had grown up on these stalks; the vines were allowed to remain on the stalk. At 15,000 plant population per acre these plants should produce over 150 bushels per acre.

Table 2--Interception storage by corn plants application rate ten inches per hour

Plant number	Lot number	Mass before wetting	Mass after wetting	Gain	Ears per plant
		lb	lb	lb	
1	A	2.50	2.75	0.25	1
2	A	2.55	2.85	0.30	1
3	A	2.70	3.00	0.30	1
4	A	2.85	3.15	0.30	1
5	A	2.85	3.35	0.50	2
6	A	3.05	3.40	0.35	1
7	A	3.30	3.60	0.30	2
8	A	3.40	3.85	0.45	2
9	A	3.50	3.85	0.35	2
10	A	3.90	4.3	0.40	2
11	A	4.20	4.6	0.40	2
Mean residual storage				0.35	
Mean transitory storage				0.27	
21	B	3.45	3.75	0.30	2
22	B	3.50	3.85	0.35	2
23	B	3.50	3.90	0.40	2
24	B	3.95	4.35	0.40	2
25	B	4.15	4.55	0.40	2
26	B	4.20	4.65	0.45	2
27	B	4.30	4.65	0.35	2
28	B	4.55	4.90	0.45	2
29	B	4.90	5.35	0.45	2
30	B	5.85	6.30	0.45	2
Mean residual storage				0.40	
Mean transitory storage				0.33	
Mean residual storage lots A and B = 0.38 lb[a]					
Mean transitory storage lots A and B = 0.30 lb[b]					
51	C	1.35	1.65	0.30	1
52	C	1.40	1.65	0.25	1
53	C	1.65	1.90	0.25	1
54	C	1.70	2.00	0.30	1
55	C	1.80	2.00	0.20	1
56	C	1.90	2.15	0.25	1
57	C	2.15	2.45	0.30	2
58	C	2.20	2.50	0.30	2
59	C	2.20	2.50	0.30	2
60	C	2.30	2.55	0.25	2
61	C	2.40	2.65	0.25	2
62	C	2.45	2.80	0.35	2
Mean residual storage lot C = 0.28 lb[c]					

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[a] Or 0.025 inch (assuming corn population of 15,000/acre).

[b] Or 0.020 inch (assuming corn population of 15,000/acre).

[c] Or 0.018 inch (assuming corn population of 15,000/acre).

area impermeable so that the stem flow can be collected.

A more direct method of measuring residual storage is to determine the amount of water on individual corn plants by weighing before and after a rain. Extending data on individual plants to a field of corn plants undoubtedly involves possibility of error. For instance, storage due to leaves of adjacent plants touching each other has been observed, but the amount stored is very small. Leaf movement during a rain lets it move on down the plant. This effect is probably more than compensated for by the canopy effect of leaves over potential storage reservoirs.

The following procedure was used to obtain maximum values for transitory and residual storage, and the time required for transitory storage to reach the ground. One lot of large, mature, green, corn plants was selected for the initial tests. The plants were cut off about two inches above the ground and transported to the laboratory. The plants were cut a few days after a rain so that soil moisture was high and turgor of the plants was good. Care was taken to keep transpiration losses to a minimum. Each plant was weighed, then placed upright under an artificial rain of about ten inches per hour for periods ranging from 40 min to an hour. Each plant was removed in turn, placed upright in a weighed container and allowed to drain for four minutes then reweighed. The water held on the plants after they were allowed to drain was obtained by difference. The container holding the drainage was again weighed and the mean amount of water draining from the plants was computed.

The period between the time the plants were cut and application of artificial rain ranged from two to four hours. The average transpiration loss during this period was 24 gm/hr. This loss was too low to cause noticeable wilting of the plants. Transpiration subsequent to weighing was determined to be negligible.

A survey of the area disclosed some fields with larger corn plants than those selected in lot A so lot B was selected. It also seemed possible that mature, field-dry corn could absorb a large amount of water, especially, if the rain were of long duration, therefore, lot C consisting of mature, field-dry corn was selected. The same procedure as outlined for lot A was followed with lots B and C, except, transitory storage was not measured on lot C.

Lot D was selected to investigate the effects of a lower application rate and a shorter time of application. Lot D was quite similar to lot B. The same procedure followed on lots A and B was used on lot D except that a two inch per hour application rate was used and the amount of residual storage was checked at both ten-minute and one-hour intervals.

Results and discussion--A description of the lots of corn plants is given in Table 1. Interception storage data for individual corn plants are given in Tables 2 and 3. The mean amount of residual storage for all green, mature corn plants listed in Table 2 (application rate ten inches per hour) is 0.38 lb. Assuming a corn plant population of 15,000 plants per acre, the calculated residual storage would be 0.025 inches. Similarly, the means for the green, mature corn plants listed in Table 3 (application rate two inches per hour) is 0.33 lb or 0.022 inch for the ten-minute application and 0.36 lb or 0.024 inch for the one-hour application.

We expected the mean residual storage for lot D to be greater than that for lots A or B due to the greater mean weight of the plants. This was not the case. Therefore, we suspect that either the lower application rate or the smaller total amount of rainfall caused the low storage values. A further decrease was noted where application was at two inches per hour for only ten minutes. This points to the smaller amount of water applied as the cause for decreased storage. Some storage pockets on the plant, possibly the tip of the ear, apparently fill only as chance drops enter.

The mean residual storage for the brown, field-dry corn of lot C is 0.28 lb or 0.018 inch as given in Table 2. This value is appreciably lower than those for the green, turgid corn of lots A and B. Also, Table 3 shows that the mean residual storage for green, mature corn is reduced from 0.36 lb to 0.34 lb after being allowed to dry for 27 hr. It appears, therefore, that the net effect of drying is to cause a decrease in the residual storage. This has been observed to be due largely to the ears turning down.

The data show that the effect of an erect ear on storage of water is greater than its weight would indicate. In the mature stage, the husk on the ears may be open, thereby forming pockets with appreciable storage capacity. In lot A, the plants with two erect ears have stored 0.10 lb more than those with one ear while in lot C, where the ears were mostly down, the plants with two ears stored only 0.03 lb more than those with one ear. The data for lot D also indicate the importance of the ear on residual storage. After allowing the corn plants to dry for 27 hr, part of the ears

Table 3--Interception storage by corn plants with application rate two inches per hour

	Plants turgid					Plants air dried			
Plant number[a]	Mass before wetting	Mass after ten m application	Gain	Mass after one hour application	Gain	Mass before wetting	Mass after one hour application	Gain	Position of ears
	lb	lb	lb	lb	lb	lb	lb	lb	
72	3.72	4.00	0.28	4.00	0.28	2.83	3.08	0.25	1 down
74	3.77	4.00	0.23	4.00	0.23	2.87	3.18	0.31	erect
76	3.90	4.20	0.30	4.25	0.35	3.05	3.44	0.39	erect
78	4.19	4.60	0.36	4.65	0.41	3.51	3.85	0.34	1 down
80	4.20	4.55	0.35	4.55	0.35	3.19	3.56	0.37	erect
82	4.36	4.70	0.34	4.70	0.34	3.46	3.83	0.37	1 down
84	4.37	4.70	0.33	4.75	0.38	3.29	3.70	0.41	erect
86	4.53	4.90	0.37	4.95	0.42	3.40	3.72	0.32	both down
88	4.75	5.10	0.35	5.15	0.40	. . .	. . .	. . .	. . .
90	4.87	5.25	0.38	5.35	0.48	3.85	4.18	0.33	both down
Mean residual storage			0.33		0.36			0.34	
Mean transitory storage = 0.05 lb									

[a] These plants are from lot D and are all two-eared with ears erect. The odd numbered plants were used to check the transpiration rate.

turned down and part were still erect. In each case where the ears were still erect the residual storage increased slightly and in each case with one exception where one or both of the ears turned down there was a small decrease.

It should be noted that differences between the mean residual storage values are small despite the wide variation in test conditions. The range for all lots and all test conditions is from 0.018 inch to 0.027 inch. A high degree of accuracy was not attempted as most weighings were only accurate to plus or minus 0.03 lb.

The mean amount of transitory storage per plant for lots A and B was 0.30 lb as listed in Table 2 (application rate ten inches per hour). Again assuming a corn population of 15,000/acre, the mean transitory storage under these conditions was 0.020 inches. Similarly, the mean transitory storage for lot D was 0.05 lb or 0.003 inch as given in Table 3 (application rate two inches per hour). These figures are slightly low due to loss of water during the fraction of a second it took to transfer each corn plant to the container receiving the drip and stem flow.

The time allowed for each plant to drain was five minutes on the first few plants but it was noted that drainage after four minutes was negligible so the time was reduced to four minutes. Even during the fourth minute the rate dropped to a slow drip. In other words, the maximum effect of transitory storage could only be a delay of less than three minutes in the time that 0.02 inch of rain reached the ground.

Application of data--It should be noted that the several components of interception have different applications. From an erosion standpoint the total amount of rain intercepted is an important consideration because all intercepted rainfall has lost much of its impact energy. The drip and stem flow would not be as erosive as rainfall reaching the ground directly.

Ground rainfall rates and amounts are necessary in many hydrologic calculations; in these cases, drip and stem flow are just as significant as rainfall reaching the ground surface directly. Any measurement of ground rainfall, consequently, must include the drip and stem flow, but due to difficulties involved in measuring stem flow, it is simpler to obtain ground rainfall by subtracting from total rainfall the portion of interception that is retained on the plants.

Knowledge of ground rainfall rates, and hence interception storage rates, is necessary in determining infiltration rates by analysis of a hydrograph. If interception storage rates are assumed too high during any portion of a hydrograph, it will lead to an infiltration rate curve that is too low, unless compensated for by ground storage rates lower than the true values.

When total ground rainfall or infiltration is needed over a period of time, for example, on an annual basis, the assumption of values for residual interception storage which do not take stem flow into account can lead to estimates that are much too low. The error would be cumulative with each rain. Accurate interception storage values are expecially important in the field determination of the amount of rainfall available to growing crops. If the estimate of the amount retained on the foliage is, for instance, one-half an inch too high for a particular rain, only a few such rains may result in a value several inches too low in the amount of rainfall available to the crop during a season.

Conclusions--For a rain sufficient to cause runoff, we consider the use of 0.02 inch residual interception storage to be applicable for a good stand of corn at any time subsequent to tasseling. Further, we consider this amount to have been stored by the time ground runoff begins.

When total ground rainfall is needed over a period of time, the assumption of values for residual interception storage which do not take stem flow into account will lead to estimates that are much too low. The residual interception storage value as determined above should serve to limit errors to less than 0.02 inch for each rain. The accurate determination of ground rainfall over a period of time must depend on further evaluation of residual interception storage values for the smaller rains and various stages of growth.

The net effect of transitory interception storage is so small that the error involved in using rainfall rates instead of ground rainfall rates is neglibible for most hydrologic purposes.

It is noteworthy that while the magnitude of the interception storage values obtained is practically insignificant, the fact itself is quite significant because of its differing so widely from the generally accepted concept that large portions of rainstorms are held in interception storage.

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THE USE OF RADIOISOTOPES FOR THE MEASUREMENT OF THE WATER EQUIVALENT OF A SNOW PACK

R. W. Gerdel, B. Lyle Hansen, and W. C. Cassidy

Abstract--A radio-telemetering snow gage has been designed which uses the principle of absorption by water (liquid or solid phase) of the gamma emissions from an artificial radioactive isotope. The radioactive, radio-telemetering snow gage has been used successfully to measure up to 55 water equivalent inches of snow by counting the pulses from a suitably located Geiger-Muller tube. These pulses have been successfully transmitted and relayed by radio. Further progress is dependent upon the development of a reliable radio communication system for use with the radioactive snow gage in remote high mountain areas.

Introduction--Data from both the precipitation gage network and the snow surveys in the high mountains of the West are limited by the inaccessibility of many important watersheds and the hazardous travel conditions during winter. The development of a reliable and economic method for the automatic measurement and telemetering of precipitation and the amount of water stored in the mountain snow packs appears to be the only feasible means of improvement in the quality and quantity of the data required for the forecasting of stream flow and the operation of the multipurpose hydrologic projects in the West. The use of telemetering systems for hydrologic data may be considered as analogous to the use of the radiosonde and the application of radio telemetered upper air data to weather forecasts. The artificial radioactive elements which have become available in recent years appear to be adaptable to the construction of a thickness gage which could be used to measure the water equivalent of the snow cover and the volume of precipitation. These irradiated elements, like such natural radioactive substances as radium may emit alpha, beta, or gamma rays. Generally speaking, the gamma rays are about 100 times more penetrating than the beta rays and 10,000 times more penetrating than the alpha rays. Only the gamma rays would appear to have sufficient energy to penetrate a deep snow pack.

Theoretical approach to the problem--The gamma rays or electromagnetic radiations emitted from a radioactive material are discrete photons or quanta characterized by their energy which may be expressed in millions of electron volts (mev). The intensity of such radiation after passing through a uniformly absorbing medium depends upon the intensity of the incident radiation, I_o, and the thickness of the medium, x, in accordance with Lambert's law, $dI = \mu I dx$, which states that the amount of radiation absorbed is proportional both to the change in thickness, dx, and to the intensity, I, and to a proportionality factor, μ, which is a characteristic property of the medium and usually referred to as the linear absorption coefficient. It is the reciprocal of the thickness, x, that will reduce the intensity of the incident radiation to 1/e of its original value. The linear absorption coefficient, μ, when divided by the density of the substance, ρ, produces the mass absorption coefficient, μ/ρ, which is a constant for the substance regardless of its state of aggregation, that is, as gas, liquid, or solid. The measured intensity of the radiation which passes through snow, therefore, may be expressed in terms of the water equivalent of the snow pack regardless of its density or the variations of density within it. However, there are three conditions to be met before the above statement is applicable. (1) The radiation must be essentially monochromatic because the absorption coefficient is a constant only at a specified wavelength. (2) The beam of radiation which is to be measured must be collimated to reduce excessive scattering of the radiation which might otherwise produce an apparent variation in the absorption coefficient and this variation would probably be a function of the density and possibly other physical phenomena of the snow pack. (3) Cosmic rays will comprise a part of the measured radiation and this cosmic ray "background" must be known or determined and subtracted from each measurement unless the intensity is much greater than the background.

Preliminary investigations--Some preliminary experiments indicated that the gamma emission from about 20 millicuries of radiozinc could be detected above the cosmic background through 36 inches of water with a small survey type meter equipped with a Geiger-Muller tube. Investigations on the practical application of this phenomenon to the measurement of the water equivalent of a snow pack, were undertaken at the Central Sierra Snow Laboratory operated jointly by the Corps of Engineers and the Weather Burear near Donner Summit, California. Thirty millicuries of radiozinc were installed beneath a forty-inch snow pack of 50 per cent density in the spring of 1948. A Geiger-Muller tube and portable survey meter were suspended above the snow pack and by suitable modification, the meter was connected to a recording potentiometer in the snow laboratory. A strip chart record of the depletion of the snow pack was obtained for a period of one week. This was probably the first record of snow melt, in situ, ever obtained without disturbance of the snow pack by removal of samples.

The successful results obtained in the preliminary studies warranted the expansion of this project and the development of a practical radio-telemetering snow pack gage.

Snow pack gage--After examining the characteristics of all available sources of gamma radiation, the radioisotope, Cobalt 60, was selected as the most suitable for the purpose of this instrument. Reasons for the choice of this radioisotope were: (1) Its gamma radiation of 1.1 and 1.3 mev is essentially monochromatic and it has sufficient energy to give a usable absorption coefficient. (2) It has a high activity. It is available in 20 millicuries units, (1 millicurie equals 3.7×10^7 disintegrating atoms per second) and each disintegrating atom emits 2 photons. Each unit emits a total of 15×10^8 gamma photons per second. This is five times the activity of Zn 65 which was used in the preliminary experiments since the Zn 65 is supplied in 15 mc units and produces on an average only one photon for two disintegrations. (3) It has a long half-life, 5.3 years, as compared with 250 days for Zn 65. This means that the decay correction will be small within a snow season and that the cost of a unit is spread over several seasons.

A Geiger-Muller tube was chosen to measure the intensity of the radiation because of its simplicity and the fact that the photons cause pulses of electricity to flow in this type of tube. These pulses can be used to modulate or key a radio transmitter and the pulse rate, which is proportional to the intensity of the radiation, can be evaluated at the receiving end of the radio-telemetering system. In this way, one set of scaling or counting equipment and recorder can be used for a number of gages.

The self-quenching type of Geiger-Muller tube selected for this project has a limited life of approximately 10^8 counts. Although the charactersitics of the tube change as the quenching vapor is used up, this change can be compensated for by automatic superimposition of a test pulse rate from a Beta Standard upon measured gamma emissions for a portion of the time of transmission. For remote, unattended operation the simplicity of the Geiger-Muller tubes warrants their use in spite of their low efficiency of 0.5 per cent for measurement of gamma rays. Their life is long enough to last through a snow season with a reasonable safety factor and adjustment for any change in characteristics can be made by means of the Beta Standard.

A drawing of the field installation as used in the Central Sierra Snow Laboratory during the 1948-1949 snow season is shown in Figure 1. Two units of the radioisotope, Co 60, were enclosed in a heavy lead shield which was installed flush with the soil surface. The shield was so designed that the collimated beam produced at 15 ft was only slightly greater in diameter than the effective length of the Geiger-Muller tube.

The Geiger-Muller tube was suspended at the end of a cross arm 15 ft above the radioisotope and carefully centered within the collimated cone of emissions. It was wired to a radio transmitter which was mounted on the vertical support. A standard radio receiver was located at the laboratory headquarters slightly over one mile from the field station. The pulses from the Geiger-Muller tube, detected by this receiver, were suitably amplified and shaped to permit operation of a standand decimal type scaler. At different times during the season the field station was actuated by a programming device incorporated in the transmitter and by an "on call" transmitter receiver operated from the laboratory headquarters.

In subsequent experiments conducted furing the summer of 1949, a Beta Standard consisting of less than one-sixth mc of Strontium 90 was included in the housing with the Geiger-Muller tube. Normally the Beta emissions were absorbed by a moveable metal shield between the standard and the Geiger-Muller tube. Automatic operation of this shield to permit superimposition of a standard count rate was effected by means of a solenoid actuated during a portion of the measuring period by the "on call" or programming device.

Calibration--A typical calibration curve showing the counts per minute above cosmic background as obtained with the above field installation, and a similar installation at the laboratory headquarters, is presented in Figure 2. Most of the points on this curve were derived from calibration experiments conducted on snow of various densities. Some additional points which are not identified in this figure were obtained at the end of the snow season by introducing measured depths of water in a tank installed between the radioisotope and the Geiger-Muller tube.

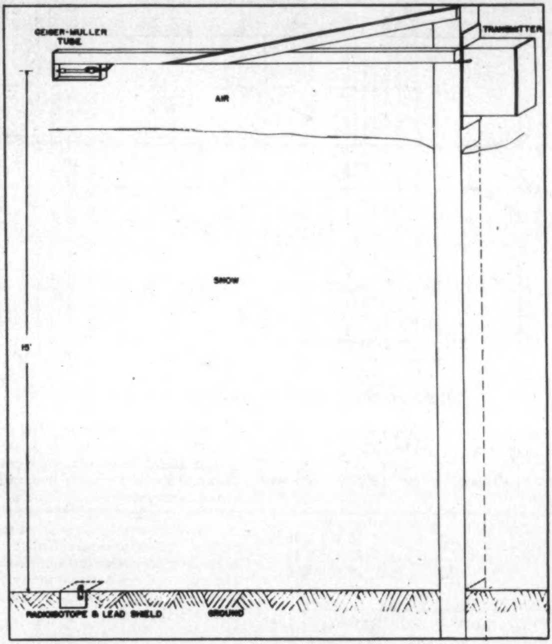

Fig. 1--Drawing showing field installation radioactive telemetering snow gage

For all snow packs up to 52 inches of water equivalent the equation $I = I_o e^{-\mu x}$ defines this curve. The linear absorption coefficient, μ, of 0.151/in obtained in these experiments is in reasonably close agreement with the accepted value of 0.160/in [LAPP and ANDREWS, 1948] for the absorption of photon energy of 1.2 mev by water. The difference between the measured and accepted value for the absorption coefficient may be attributed partly to the unequal proportions of 1.1 and 1.3 mev energies of Co 60 reaching the Geiger-Muller tube, partly to scattering of the gamma rays resulting from imperfect collimation of the photon beam and partly to the failure to apply a correction for the counts lost due to the finite resolving power of the tube. Experiments with heavier shielding have shown that the more perfect the collimation the more closely μ approaches the accepted value of 0.160/in. For precision measurements with a self-quenching tube of the type used in these experiments it is usual to apply a correction factor of 0.5 per cent per thousand counts. If such a correction factor had been applied to the data plotted in Figure 2 there would have been a closer agreement between the measured and accepted value for the absorption coefficient for water. Since the measured absorption coefficient remained a constant within the 52-inch water equivalent depth of snow at the experimental site, the application of a correction factor was not considered necessary.

The probable error of the counting rate which was computed for grouped data used in plotting Figure 2 is presented in Table 1. A mean value for the cosmic background was subtracted from the total counts before computing the probable error. For a mean background value of 140 cpm for ten one-minute intervals, the probable error was 2.5 per cent. The greater portion of the error in measurement of the water equivalent of the snow by this procedure appears to be attributable to the fluctuation of cosmic ray background.

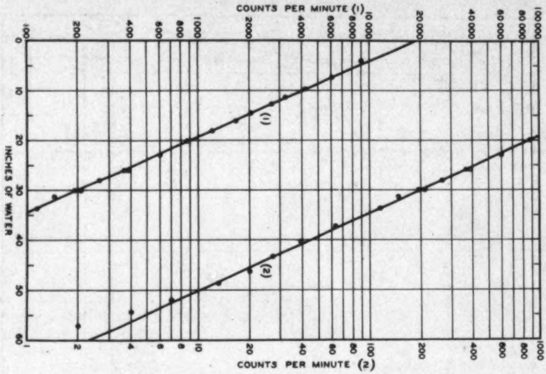

Fig. 2--Calibration curve for radioactive snow gage

Table 1--Probable error of the counting rate and equivalent inches of water[a]

Counting rate[a]	Probable error[b]	Water equivalent
CPM	pct	in
20	3.5	45.5 ±1.59
300	1.5	27.0 ±0.40
3000	0.5	12.0 ±0.06
9000	0.3	4.7 ±0.01

[a] [see JAFFEY, KOHMAN, and CRAWFORD, 1943]

[b] Median for group; variation within group did not exceed ten per cent of median; duration of counts varied from two minutes for high counting rates to ten minutes for low counting rates.

[c] Per cent probable error = $0.6745/\sqrt{M}$; M = (total counts - background).

Radio telemetering--The pulses from the Geiger-Muller tube transmitted from the field station could be consistently detected on a standard radio receiver operating on 2350 kc at the laboratory headquarters. From the number of pulses registered in a unit period of time on a scaler connected to the radio receiver, the water equivalent of the snow pack at the remote field station could be determined. The pulses were satisfactorily relayed at this frequency to Sacramento, California, a distance of 70 air line mi.

Further development of the radio-telemetering system will require the use of frequencies within the 169-173 megacycle range which are available for a hydrologic network. Practical use of the radioactive radio-telemetering snow gage will depend upon the development of a reliable radio transmission link, operating within the above frequencies, between the remote radioactive snow gage and the scaler located in a central office. Such a transmission link must operate without attendance at the field station for a period of seven to nine months. Some preliminary investigations indicate that a satisfactory radio-telemetering system will not be difficult to design or construct nor will the cost be prohibitive.

Summary--The use of a radioisotope as a measuring medium will eliminate to a considerable degree the complicated actuating mechanism associated with similar commercial devices such as telemetering precipitation and water level indicators. With little modification the radiation absorption principle of the snow gage may be applied to the measurement and telemetering of other pertinent hydrologic data. Theoretical considerations supplemented by a few limited experiments indicate that this principle may be applied to the measurement of water levels, ground-water elevation, in situ, without construction of wells, and change in the moisture content of soils.

Improved types of self-quenching Geiger-Muller tubes which have become available recently should insure a reliable operating life under field conditions and make the radioactive absorption gage principle at least as reliable, and certainly more versatile, than any existing method for obtaining hydrologic records from remote stations. The development of Geiger-Muller tubes or reliable scintillation tubes of even slightly greater efficiency than those presently available, would greatly extend the scope of this type of telemetering system.

The possible health hazards inherent in any use of radioisotopes have been recognized. In remote mountain areas for which the radioactive snow gage was developed such hazards are almost nonexistent. In more populated areas, suitable shielding and fencing of the area may be required. Precautions similar to those taken by any electric power company for the protection of transformer substations should be sufficient to protect the inquisitive visitor.

Acknowledgments--The project for development of an automatic radio snow measuring instrument is a Civil Works Investigation of the Corps of Engineers. Development of the use of radio isotopes for this purpose was undertaken jointly by the Corps of Engineers, Department of the Army, and the Weather Bureau, Department of Commerce. The Zinc 65 used in these investigations was procured by the U. S. Weather Bureau and the Cobalt 60 by the Corps of Engineers, Sacramento District, on allocation from the Isotopes Division, U. S. Atomic Energy Commission. The Strontium 90, for the Beta Standards was obtained by the Corps of Engineers, Sacramento District, from the United States Radium Corporation on allocation from the Isotopes Division, U. S. Atomic Energy Commission.

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CAUSES OF MOTIONS IN THE EARTH'S CORE

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Abstract--The geomagnetic secular variation can be interpreted as the expression of motions in the fluid, electrically conducting core. The investigation of the possible causes of such motions shows that thermal convection provides the only satisfactory mechanism for the generation of motions. If five to ten per cent as much radioactivity as that contained in the crust is concentrated in the central region of the Earth inside the Lehmann-Gutenberg discontinuity, convection in the fluid core will take place. A study of the lunar perturbations of the core, tidal, precessional, and others shows that these effects should be negligible in the generation of motions as compared to the thermal effects.

Introduction--In recent years there has been a growing amount of evidence that the phenomena of terrestrial magnetism are caused by electric currents flowing in the Earth's metallic core (disregarding, of course, those variable fields whose origin can be traced to the ionosphere). The most cogent reason for postulating this origin is found in the time-scale of the secular geomagnetic variation, involving periods of the general order of a few centuries or even decades; these must of necessity be assumed to originate in the fluid rather than in the solid part of the Earth; that is, in the core rather than in the mantle.

The mathematical problems involved in such a theory and some of its physical implications have been treated in a number of earlier papers by the writer [ELSASSER, 1946, 1947], and by BULLARD [1948, 1949]. In this view the existence of electric currents in the core must be understood as a result of mechanical motions in the fluid constituting it. Such motions generate electric currents or, rather, they are able to amplify such currents from small beginnings to a terminal value dependent on the fluid velocity according to the same electrodynamic laws which operate in conventional rotating generators of electricity. The reader who desires to become acquainted with these problems without assimilating all the mathematical arguments of the original papers may be referred to two recent publications: a semi-popular article by BULLARD [1949] giving an account of the application of the electromagnetic generator principle to the Earth, and a general review of the present state of our knowledge of the Earth's far interior compiled by the writer [ELSASSER, 1950]. Since this last paper contains a bibliography of the more recent work on the subject, some statements are used below without explicit references. The reader interested in more detail should be able to find the pertinent references in that article.

If we look at the geomagnetic secular variation from the viewpoint of the geophysicist rather than from that of the mathematician, it appears as a source of information about the Earth's interior. The only major source of such information has been seismology, and that data provided by it give a purely static picture of the Earth, whereas geomagnetism indicates motions by way of the secular variation. It is possible to estimate the magnitude of the fluid velocities involved: they are about half a millimeter per second, perhaps larger [ELSASSER, 1950]. One very simple way of obtaining this value is to observe the rate at which the secular variation displaces the lines of constant magnetic field at the surface of the Earth, but there are other other indirect methods to corroborate the result. We now have a very extensive and detailed graphical description and numerical analysis of the secular geomagnetic variation in the work of VESTINE and Others [1947] which covers the first forty years of the present century. The maps of the secular variation have an astounding similarity to weather maps, and from the viewpoint of the electromagnetic theory the analogy to weather maps is far more than a coincidence; these maps appear to reflect motions occurring in the fluid core, more precisely, in its upper parts. One of the principal problems that arises here is that of identifying physical processes that can cause mechanical motions in the Earth's core. Studies of this type have been undertaken, particularly by BULLARD [1948, 1949]; some of his arguments are very briefly outlined below, and the discussion amplified. We hope that it has been possible to cover at least the qualitative aspects of the question with a reasonable degree of completeness and to avoid omitting any of the more obvious mechanisms which can cause motions of the type that the geomagnetic secular variation puts in evidence.

If motions in the core exist at the present time, we can hardly fail to draw the conclusion that they have been going on for most or all of the Earth's life. Recent studies of "fossil" magnetism indicate that a magnetic field and its secular variation have existed during an appreciable fraction of geological history [ELSASSER, 1950]. We shall, therefore, assume that we are dealing with a process where in the main, the production and dissipation of energy is constant over times of geological length. Now fluid motions on the Earth are of two general types, being either of thermal or of gravitational origin. The circulation of the atmosphere is of the former, the oceanic tide of the latter type. This classification leaves out some rather minor causes of motions (for example, changes of salinity in the oceans) whose analogues in the Earth's core are unlikely to be effective over long periods of time. We are thus led to consider thermal convection, probably caused by a small amount of radioactive material in the core. A simple analysis shows that three different models of a convective core can be distinguished, depending on some obvious assumptions about the distribution of heat sources and of the relative heat conductivities of core and mantle. These are discussed below together with the arguments for and against them. It seems that at the present time a decision between them cannot be made. The effects of gravitational perturbations by the Moon, and to a lesser degree by the Sun, such as tides in the core, effect of the equinoctial precession, and of the slowing down of the Earth's rotation by tidal friction are also considered. Bullard found that although these effects might be adequate purely from the viewpoint of energy, there exists no dynamic mechanism whereby such perturbations can generate internal motions of suitable magnitude in the core. Bullard's arguments did not seem quite comprehensive to the w writer, and therefore, the study outlined in the last section was undertaken. The result is now unambiguous: There seems to be no way of explaining motions of the magnitude observed in the geomagnetic secular variation by any of the gravitational perturbations produced by the Moon or Sun. Summarizing these results it might be said that if the interpretation of the geomagnetic phenomena as indicating motions in the core is accepted, thermal convection is almost certainly the primary process.

Constitution of the core--The traditional view of the constitution of the Earth's far interior is that a mantle of silicates surrounds the core of nickel-iron. Figure 1 shows the velocities of P-waves (longitudinal) in the core according to GUTENBERG and RICHTER [1938]; the curve given by Jeffreys differs from this in some details, but these may be omitted for the present purpose. The region (radius about 1300 km) inside the steeply rising part of the curve is designated as the central body. The region between the outer boundary (radius 3500 km) and the boundary of the central body is designated as the liquid layer; sometimes, where confusion is not likely to occur, this layer is also called the core. We shall also, for the sake of convenience, speak of the boundary of the central body, although the measurements do not indicate the presence of a sharp boundary, but rather that of a transition layer.

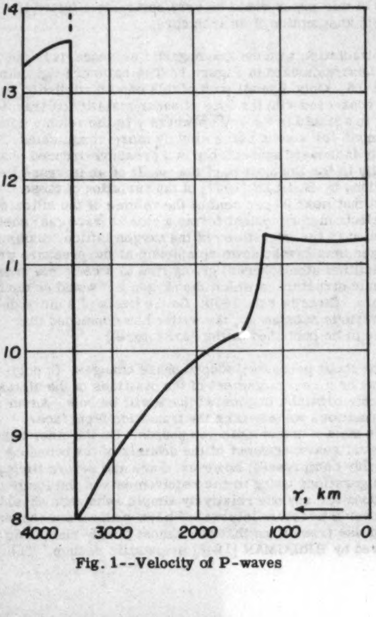

Fig. 1--Velocity of P-waves

It is a basic experience of geophysics that any discussion of the actual state of the Earth's interior cannot be entirely dissociated from the questions of dynamics, especially the problems of the history and formation of the Earth. This is patently clear, for instance, in the book by JEFFREYS [1929]. Certain speculations about the formation of the Earth have in recent years led to the idea that the Earth's core does not consist of iron. In view of the significance that such a result would have for the dynamics of the core we must discuss it briefly. The idea advanced by KUHN and RITTMANN [1941] is that the viscosity inside the Earth is so large that no chemical separation can have taken place, hence, they consider the whole interior of the Earth as constituted of undifferentiated solar matter whose main component is hydrogen. The boundary between mantle and core represents then merely a

physical change of state, no chemical differentiation. The absence of transverse waves from the core is explained by a high coefficient of absorption of these waves owing to large viscosity. This view has already been most severely criticized by Jeffreys, Eucken, and others [see Review by ELSASSER, 1950]. Its basic idea is that the viscosity not only is, but always has been, very large. Viscosity, however, is a function not only of pressure, but also of temperature. Thus in experiments of Bridgman on water near 0° C. the increase in viscosity produced by a pressure of 8000 atmospheres can roughly be canceled by a 30° rise in temperature. There is no reason whatever to believe that this behavior should change qualitatively at higher pressures and temperatures.

It is also extremely difficult to reconcile this hypothesis with the known seismological data. This has been emphasized by all its critics. The discontinuity at the boundary of the core is not only very pronounced (see Fig. 1), but also completely sharp within the accuracy of the seismological measurements which is rather high. There can be little doubt that this is a genuine phase boundary and the question remaining is whether the substances on both sides of the boundary are the same or different.

This problem has been approached lately by an interesting and novel hypothesis put forward by RAMSEY [1948]. His starting point is the observation that the inner planets have densities that cannot readily be reconciled with the present views about the formation of the solar system. The main implications of the hypothesis are geophysical, however, and they can be discussed without reference to astronomical data. Ramsey assumes that the Earth consists of homogeneous material, having essentially the composition that is usually ascribed to the mantle, namely, a simple magnesium-iron silicate such as olivine. He interprets the boundary of the core as a phase-change in this material. It follows from general principles of quantum theory that at extreme compression every substance should become an electronic conductor. When this state is approached the ordinary chemical bonds are broken down by the action of the hydrostatic pressure. The material of the core would in this view be a metallic phase of the silicates; it must, of course, also be assumed fluid in order to satisfy the seismological observations. From the viewpoint of a theory of terrestrial magnetism essentially only electrical conductivity and fluidity are required of the material in the core, so that Ramsey's hypothesis would not be in contradiction with this theory. In fact, none of the estimates in the following two sections of this paper would be very seriously affected if Ramsey's view were substituted for the current assumption of an iron core.

Although the idea of Ramsey is not in contradiction with the geomagnetic evidence, it is not easy to reconcile it with the seismological data summarized in Figure 1. The ratio of longitudinal velocities inside and outside the core is about 0.6. Only a small part of this can be attributed to the decrease of velocity that is automatically connected with the loss of shear stability on transition to the liquid state. The velocity of sound in a liquid is $v = 1/\sqrt{\gamma\rho}$ where γ is the volume compressibility, ρ density; the corresponding formula for a solid being slightly more complicated. It can hardly be assumed that the compressibility increases appreciably in a pressure-induced phase transition, so that the decrease in wave velocity is for the most part the result of an increase in density, in excellent agreement with computations by BULLEN [1947] of the variation of these quantities in the Earth. Now it is a known fact that near 90 per cent of the volume of the silicates is taken up by the negative oxygen ion whose electron complement forms a closed (rare gas) shell. The positive ions (silicon and metals) are located in the interstices of the oxygen lattice. Ramsey's hypothesis implies that this edifice of the oxygen ions breaks down completely at the pressure prevailing at the boundary of the core (about 1.4 million atmospheres) giving rise to a complete reorganization of the electronic as well as the atomic structure in which the oxygen ion would no doubt no longer play the same dominant role as before. (Note in Feb. 1950: On the basis of a more detailed study of the high-pressure behavior of various substances, the writer has concluded that Ramsey's theory is untenable. The results are to be published in the near future.)

In this connection we shall say a few words about pressure-induced phase changes. In polymorphic transitions in the solid state there may be a rearrangement of the positions of the atoms in the lattice or a reorganization of the electronic orbitals; in general there will be both. As an example of a change, primarily of the atomic positions we may take the transition from face-centered to body-centered cubic lattice, both in metals and in ionic compounds. On the other hand, in most chemical reactions there is a fundamental rearrangement of the orbitals of the bounding electrons. Once a substance has been appreciably compressed, however, thare are severe limitations to the number of possible electronic configurations owing to the requirements of the Pauli principle. The number of major phase transitions for any one relatively simple substance should be quite limited, hardly exceeding one or two over a pressure interval of a few million atmospheres. In simple substances there can apparently be phase transitions that are almost purely electronic in nature; such a case has recently been discovered by BRIDGMAN [1948] in metallic cesium. The transition occurs at 55000 atmospheres and it can hardly be interpreted as a mere rearrangement of atoms as, at this pressure, cesium has already the structure corresponding to a closest packing of spheres (face-centered cubic). A quantitatively satisfactory interpretation can be given on the basis of the assumption that this represents a change of the valence electron of Cs from a 6s-orbital to a 5d-orbital. The volume contraction at the transition point is 5.6 per cent of the volume at zero pressure. This is remarkably small since the atomic volume and compressibility of cesium are among the largest known. It is, however, in agreement with general experience of high-pressure physics which indicates that fractional volume changes become progressively smaller at hydrostatic pressure. Returning now to Ramsey's idea: The main objection to it, in the eyes of the writer, is not qualitative but quantitative, in regard to the observed increase in density of 30-40 per cent at the boundary of the core. This is far more than experience would prepare us to expect for any phase transition at very high pressures.

We shall for the present adhere to the older idea that the Earth's core consists of iron, more precisely of an iron-nickel alloy similar to that found in iron meteorites (average of about nine per cent nickel). This view is an agreement with all the seismological data and seems corroborated by a vast amount of geophysical evidence. It might be said, parenthetically, that if the Earth's core should not consist of iron, the meteorites can hardly be the fragments of a former planet, as is often assumed. Since the origin of the meteorites is still an open question, this does not, however, constitute an argument for or against any theory. It is sometimes thought that the core consists of iron, but that not all the metallic iron of the Earth has been concentrated in the core, some of it having remained interspersed in the mantle. This is altogether possible, but the seismic velocities in the mantle are so high that its iron content is probably not more than a moderate percentage.

There is evidence from the geomagnetic secular variation that the core is in motion. If this motion has been going on during most or all of the Earth's life, it is likely that in the liquid layer chemical separation has become practically complete; that is, the liquid layer must form a phase that is very near to chemical equilibrium (corresponding, perhaps, to a mean pressure between top and bottom of the layer). Such separate phases as can be precipitated in bulk should have become concentrated long ago either at the top or at the bottom of the liquid layer. There is no evidence of an additional stratification near the boundary of the core, as Jeffreys has pointed out; we may perhaps conclude that the primeval separation into crust, mantle and core has been quite effective.

It is well known that some of the lighter elements and also the relatively light oxides and silicates of certain elements, "lithophil" elements of GOLDSCHMIDT [1937] are strongly concentrated in the crust. It is of interest to inquire into the possibility of a concentration of relatively heavy elements in the central body. This has an obvious bearing upon the disbribution of the radioactive elements in the Earth. Measurements of recent years have fully corroborated the old idea that the radio-elements are strongly concentrated in the crust; the radioactivity found in ordinary crustal rocks is much reduced for basic rocks, and ultrabasic rocks are almost completely without radioactivity as compared to the crustal average. This concentration of the radioelements is no doubt due to the ability of the parent elements, U and Th to form relatively light compounds. The lowest oxides, however, UO_2 and ThO_2, have densities near 10 (under laboratory pressure), hence, they are specifically heavier than iron. It may be, therefore, that part of the original radioactive compounds is concentrated in the crust and another part in the central body. If the central body is solid, these radio-elements may be concentrated near its surface, if they arrived there after the solidification.

Although it is not of immediate consequence for our subject, we might discuss here a few ideas about the constitution of the central body. It is very unlikely that the central body is made up of elements or compounds heavier than iron. If we study the estimate of GOLDSCHMIDT [1937], of the cosmic abundances of the heavier elements, we find that the abundances decrease with extreme rapidity in the section of the periodic system between the iron group and the palladium group. From there on the average abundances remain fairly constant up to the heaviest elements (the abundances are, however, so variable from element to element that such regularities appear only on a logarithmic plot). The abundance of the heavy elements is about $10^{-6 \pm 1}$ of that of iron. Counting about 50 such elements, the aggregated number of atoms would be about 5×10^{-5} of this number of iron atoms. This, clearly, is not enough to fill the volume of the central body which is five to six per cent of that of the entire core. The elements, on the other hand, which stand in the periodic system in the neighborhood of the iron group are mostly metals. Moreover, since the metals of the iron group represent a minimum of atomic volume relative to their neighbors, it is rather unlikely that the other metals should form a separate body underneath the liquid iron phase; if they are not oxidized they are likely to be alloyed with iron, but their combined abundance is probably too small to affect appreciably the basic iron-nickel constitution of the liquid core.

Hence the conclusion, first drawn by Bullen, that the central body consists of iron and nickel. Bullen has pointed out that the rise in seismic velocity shown in Figure 1 is just what one should expect in the transition from the liquid to the solid phase of the same substance. This seems an entirely likely explantion for the central body. As has been pointed out above, there exists at high pressures the possibility of an electronic type of phase transition similar to that observed in cesium. Our present knowledge of the electronic structure of iron makes it almost certain that such a transition must occur somewhere in the pressure range of millions at atmospheres, but we cannot as yet indicate quantitatively the pressure required. The valence electrons of iron would change from 4s-orbitals to 3d-orbitals. At present we cannot say which of these two possible types of phase transttion occurs at the boundary of the central body. If the electronic phase transition occurs it might also be accompanied by solidification. Although we have been led to think that the central body consists mostly of iron, this does not contradict the assumption that such small amounts of relatively dense compounds that have been formed will have gathered inside it or near its boundary; in particular, we might expect a concentration of radioactivity there.

Convective models--Thermal convection occurs in the core when the transport of heat radially outwards, forced by the physical condition of the medium, exceeds the heat transport by thermal conduction alone. The material of the fluid layer should be very nearly in a state of chemical equilibrium as pointed out before. Once such a condition prevails, uniformity of the convective layer will be maintained by the mixing action of the motion itself, which is comparatively intense if conceived of as extending over periods of geological length. Hence it is improbable that the motion is caused by any variation of properties within the liquid layer itself; it will instead be related to the boundary conditions imposed upon the liquid. This consideration leads us readily to distinguish three typical mechanisms of thermal convection. First model: The heat supplied to the lower boundary of the liquid layer exceeds the amount that can be carried away from there by conduction alone. To simplify the model we may assume that this heat remains in the core, that is, the radial flow of heat in the adjacent mantle is small compared to the transport in the core itself. Second model: The heat flow in the mantle adjacent to the core is in excess of the purely conductive heat transport in the core itself. We shall assume in this case that the heat supply at the lower boundary of the core is small, or at least not large enough to cause convection by itself. Third model: The heat supplied at the lower boundary is carried to the upper boundary by convection, and is then carried away by the mantle.

We can express these models quantitatively as follows. Let F be the total radial flow of heat per unit cross section, $F = f_o + f$, where f_o is the conductive, f the convective flow. Let h be the amount of heat that is gained (h positive) or lost by unit volume of the liquid. We have the continuity condition

$$(1/r^2)\, d\,(r^2F)/dr = -h \qquad (1)$$

We are here only interested in a simple survey and we shall, therefore, assume h constant, independent of depth in the layer. We let r_1 and r_2 be the radius of the lower and upper boundary, respectively.

First model: We assume $F = m_1$ to be the amount of heat per cm^2 per second available at $r = r_1$. If the heat is not produced at the boundary proper by radioactive substances concentrated there, but comes from the interior of the central body, certain complications arise which we shall disregard here. We assume that the core is thermally insulated in this model, that is, $F = 0$ at $r = r_2$. We then find

$$f = m_1(r/r_1) \cdot [(r_2/r)^3 - 1]/[(r_2/r_1)^3 - 1] - f_o \qquad (2)$$

Second model: The volume of the central body is only a small fraction of that of the core; neglecting the thermal contribution of the central body we may set $F = 0$ at $r = r_1$. If $F = m_2$ at $r = r_2$ represents the heat lost to the mantle, we find

$$f = m_2(r/r_2) \cdot [1 - (r_1/r)^3]/[1 - (r_1/r_2)^3] - f_o \qquad (3)$$

Third model: We assume $h = 0$, the heat passes through the liquid layer without changing the thermal state of the latter. Then we have

$$f = m_1(r_1/r)^2 - f_o = m_2(r_2/r)^2 - f_o \qquad (4)$$

In the first model the maximum convective activity is at the lower boundary, and the maximum is rather pronounced. In the second model there is a less steep maximum at the upper boundary. If the convective transport is large compared to the conductive one, we may drop f_o. In Figure 2 the curves are shown corresponding to $f_o = 0$. The scale factors have been adjusted so that the flow functions are equal midway between the boundaries of the liquid layer. The third model which combines features of the two others is perhaps not particularly interesting at the present crude stage of the analysis; we shall therefore confine ourselves to the alternative between the first two models.

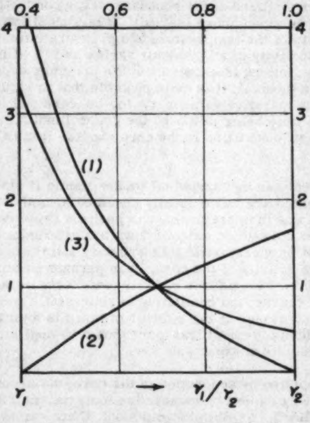

Fig. 2--Flux as function of depth

In order to obtain numerical results we first estimate the conductive flow, $f_o = \kappa\tau$ where κ is the heat conductivity of iron, τ the temperature gradient in the liquid layer. Such estimates have been made by BULLARD [1948, 1949] and we shall follow him in many details. It is rather simple to obtain a figure for κ. According to the Wiedemann-Franz law the combination $\kappa/\sigma T$ (where σ is the electric conductivity, T absolute temperature) is a universal constant for all metals. Since, however, σ is inversely proportional to T at high temperatures, it follows that κ is temperature independent, in fair agreement with observation. It so happens that the laboratory value for iron, $\kappa = 0.18$ cal/cm sec deg. agrees with the value of σ for the core estimated by Bullard and the writer, provided one takes Jeffreys' estimate, $T = 10000^o$ for the temperature of the core (which is in the nature of an upper limit). There is no reason to believe that the theoretical principles which guarantee the Wiedemann-Franz relation break down under the conditions of the core, so that the estimate of κ should be reasonably reliable.

Next we may try to estimate the temperature gradient τ. Since the core is in a state of convective agitation, it follows from known principles that the gradient must have the adiabatic value. The adiabatic temperature gradient is connected with the volume coefficient of thermal expansion α by a thermodynamical identity,

$$\tau = g\alpha T/C_p \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

(g gravity, C_p specific heat). Bullard has estimated τ for the top layers of the core, his value of α being obtained from a theoretical expression given by MOTT and JONES [1936]. He finds that α should decrease from the laboratory value for iron, 34×10^{-6}, to 9×10^{-6}. Taking g = 900 near the top of the core [BULLEN, 1947] he finds $\tau = 1.1^\circ$/km. The value at the lower boundary of the liquid should be lower since g decreases in the layer to about half the above value, and α is also bound to decrease. Furthermore, T = 10000° is an upper limit, and the actual value might be somewhat lower. We, therefore, shall take $\tau = 0.3^\circ$/km at the lower boundary, which yields for the conductive flow there, $f_o = 5.4 \times 10^{-7}$ cal/cm^2 sec. In order to realize the first model we must now assume that the actual heat supply at the lower boundary exceeds this value; let us for instance, double it: $m_1 = 2f_o$.

We inquire now into the sources of heat needed for this model. Some simple calculations will show that chemical processes would not give a supply sufficient for the lifetime of the Earth; the simplest assumption is that of radioactivity. We may compare $m_1 = 1.1 \times 10^{-6}$ with the radioactive heat developed in the crust. It is well known that a large part of the heat flow in the crust is due to radioactivity, and for an order of magnitude estimate we might use the measured mean heat outflow from the crust [BIRCH, 1942]. This is 1.3×10^{-6}, very close to the above value of m_1. Hence, the radioactive content of the central body as compared to the crust should be roughly in the ratio of their surface areas, that of the central body being about five per cent of that of the crust. We see that the convective model of the first type must operate provided a modest percentage of the radioactive material of the Earth is concentrated near the center of the Earth. There seems to be no particular difficulty in making such an assumption. If this radioactive material is thought of as spread through the central body, its concentration per unit mass is considerably less than in crustal igneous rocks, but more than in iron meteorites.

We now proceed to study the conditions at the outer boundary of the liquid layer. This means that we must determine the heat conduction in the mantle near the core. The extrapolation from laboratory conditions to those prevailing in the Earth is here beset with extreme difficulties. As a typical example of the substances composing the mantle we may take olivine. If the Adams-Jeffreys model [JEFFREYS, 1929] of temperature distribution applies, the temperature gradient is given by the melting-point gradient. For olivine this is 4.7°/km under laboratory conditions [BOWEN and SCHAIRER, 1935], and this is unlikely to change by orders of magnitude as we go to the interior. The heat conductivity is about 1.2×10^{-2} for olivine type rocks [BIRCH, 1942] giving roughly $m_2 = 0.6 \times 10^{-6}$ if laboratory conditions were realized at the boundary. (The conductive flow in the adjacent core is 2×10^{-6} if $\tau = 1.1°/km$, as given above is used). It is almost impossible, at the present time, to make a reliable estimate of the temperature and pressure effects. Theory indicates [PEIERLS, 1928] that the heat conductivity of an insulator varies as 1/T at high temperatures, giving a very strong reduction of m_1. Nothing is known about the pressure dependence of the heat conductivity over a larger pressure interval; it is quite possible that pressure improves the heat conduction. The question of the comparative value of $\kappa\tau$ for the core and the adjacent mantle must, therefore, be left open for the time being, although the general indication is that, in spite of the less steep temperature gradient, conduction in the core exceeds that in the mantle, favoring the first model.

Bullard has pointed out that a large amount of heat can be carried off by the mantle if slow viscous convection takes place. The velocities required are those usually assumed by authors who postulate such convective flow, of the order of some mm/year. One can perhaps raise the following mild objection against this view. If cellular convection extends down to the boundary of the core, there should be distinct local differences of temperature at this boundary and these ought to be reflected in localized patterns of the convective activity of the core. It is perhaps premature to say that such patterns do not exist, but there is so far no evidence of localized effects in the secular geomagnetic variation. The argument is, of course, too qualitative to represent a weighty objection. Radiation which is of such fundamental importance in the interior of stars is a small effect at the temperatures prevailing in the Earth. Radiative heat transport should be negligible unless the opacity of the mantle could be shown to be quite unexpectedly low.

If the heat carried in the mantle exceeds that supplied by conduction in the core, the second convective model becomes operative. The heat can be supplied by radioactive material distributed throughout the mantle; this has been proposed by Bullard. As Bullard points out, there exists the difficulty that the observed radioactivity of iron meteorites is smaller than that of stony meteorites. It is, however, not necessary to assume such radioactive heat sources; the convective process will always take place if the heat conduction of the mantle is large enough, the net result being an average loss of heat of the core over the geological ages. Finally, the conditions of applicability of the third convective model are obvious and need not be detailed.

In the first model, there is a net secular rise of the temperature of the core. This can readily be calculated from the figures used and is of the order of some 100° during the lifetime of the Earth, thus without perceptible geophysical effects and not in any way alarming. Similarly, in the case of the second model one can readily adjust the figures so that the net cooling is only on the order of a few hundred degrees.

Part of the thermal energy carried by convection takes the form of kinetic energy of the fluid motion, whence it can ultimately be converted into energy of the electromagnetic field. The fraction of the energy available for this purpose is limited by the second law of thermodynamics. It has been shown [BULLARD, 1948, 1949] that the convective processes discussed above yield an ample supply of power for the maintenance of the motions and the electromagnetic generator mechanism. Hence, as far as one can see at present, the convective models are rather satisfactory. It is difficult to say which one of the three models (or which intermediary case) occurs in the Earth. Perhaps this question can best be approached by a study of the actual Earth's magnetic field and its secular variation which might permit conclusions about the character of the underlying convective pattern. It is perhaps premature to draw such conclusions at the present time.

Lunar perturbations--The study summarized below leads to the conclusion that the gravitational effects of the Moon and the Sun upon the equilibrium of the fluid parts of the Earth cannot be held responsible for the fluid motions to which we attribute the geomagnetic secular variation. In view of the negative character of this result we have omitted a number of details of the calculations. It will not be necessary to deal with the Sun separately, as the solar perturbations are appreciably smaller than the lunar ones and are, therefore, certainly negligible in this context when the lunar ones are.

We may divide the effects of the Moon into two types, the short-period perturbations, that is the tides, and the long-period or aperiodic phenomena, specifically precession and the slowing down of the Earth's rotation. We first consider the tides. The so-called equilibrium tide is defined as the static deformation of a fluid surface, keeping the Earth and Moon in fixed positions. The deformed surface must be an equipotential surface and its equation is [LAMB, 1932]

$$z\,(\partial V/\partial r) + W = \text{const} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (6)$$

where z is the radial displacement engendered by the tide, V and W are the gravitational potentials of Earth and Moon, respectively. Since $\partial V/\partial r = g$ (gravity) we have (on dropping the constant) $z = -W/g$; on inserting the value of W from potential theory

$$z = 2GMr^2/3gD^3\,(1/3 - \cos^2\theta) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (7)$$

where G = value of gravity; M and D are the mass and distance of the Moon; r radius of the fluid surface; θ is measured from a line connecting the Moon and Earth. The factor in front of $(1/3 - \cos^2\theta)$ is called the height of the equilibrium tide. It is about 55 cm for the free ocean surface. It would be about 17 cm at the boundary of the core if the Earth were liquid and about two cm at the boundary of the central body. Actually the mantle is solid and the amplitude of the lunar solid tide has been accurately measured at the surface of the Earth [JEFFREYS, 1929] and is 15 cm. JEFFREYS [1924] has computed the corresponding value for the boundary of the core and finds it to be about six cm. The actual tides are of course not static, but they cannot exceed the general mangitude of the equilibrium tide unless there is a pronounced resonance. Closer inspection shows [LAMB, 1932] that the apparent lunar period must agree very closely with a free period of the fluid Earth in order to produce a large resonance amplitude. This case is so unlikely that we may safely exclude it unless we could some day find empirical evidence for its existence.

We may take $z = 10$ cm as an upper limit of tidal displacements in the core. For the semidiurnal tide this gives a velocity of 5×10^{-4} cm/sec. One can readily show that this value is not appreciably changed if the effect of the Coriolis force upon the tidal motion is taken into account. We are, however, not interested in purely oscillatory motions as they would not give rise to the generation of an electromagnetic field, and the evidence of the geomagnetic secular variation shows [ELSASSER, 1950] that the motions are not simply oscillatory. Again, it can readily be proved that the aperiodic motions that can occur inside the closed vessel containing the core, are motions that have vorticity. Those are also the ones that will result in the generation of an electromagnetic field. It is a general principle of geophysical hydrodynamics that it is very much easier to generate fluid motions along the circles of latitude than motions in the meridional planes. We may therefore think of the aperiodic components of the motion as a flow parallel to the circles of latitude, the velocity being a function of the depth. The vorticity required appears by virtue of the fact that there is surface vorticity when a wave moves along the interface of two different media; thus there would be surface vorticity in the tidal motion at the boundary of the core. This surface vorticity will be distributed through a layer of the core by turbulent friction. In order to estimate the aperiodic component fo the motion, we set $v = v_o + v_1$ where v_o is a periodic solution of the tidal equations which is obtained from the hydrodynamic equations by omitting the non-linear term $(v\nabla)v$. The equations determining v_1 are

$$\dot{v}_1 = -(v_o\nabla)\,v_o - \nabla_p/\rho \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (8)$$

where we have omitted the Coriolis terms since we want to confine ourselves to motions along circles of latitude. If the first term on the right hand side is developed in a Fourier series with respect to the period of the tide, there will be a constant term even though v_o is periodic, because the expression is quadratic in v_o. The second term, giving the pressure variation may be considered purely periodic. If A_o is the first term of the Fourier series, we have for the aperiodic acceleration $(\dot{v}_1)_{ap} = A_o$. From this acceleration we at once get the mechanical power that this aperiodic process can deliver. There remains the substitution of the numerical value of v_o given above and some minor calculations. The final result is that any aperiodic acceleration of the fluid in the core falls short of that required by the evidence of the geomagnetic secular variation by a factor of 100 to 1000. We may safely conclude that the tidal action of the Moon upon the core is not the cause of the observed motions.

We shall next consider those effects where the lunar influence is indirect, caused by a non-uniformity of the rotation of the mantle. The most conspicuous of these is the precession of the equinoxes. Poincaré has proved [LAMB, 1932] that if the core consists of a homogeneous fluid, then the ellipticity of the boundary acts as a constraint that effectively prohibits internal motions of the fluid so that the latter follows the precession of the mantle as if solidified. Now the core is certainly not homogeneous, but contains the central body. One might ask whether the equinoctial precession of the mantle can lead to a state where the central body, considered as solid, rotates about an axis slightly different from the axis of the mantle. This would imply a tilt of the ellipsoidal boundary of the central body relative to its equilibrium position. This requires work to be done on the core since the symmetrical position represents a minimum of gravitational energy. If the energy is supplied by the Moon, the displacement of the boundary cannot appreciably exceed the equilibrium tide, as this tide represents just the work done by the Moon against the mutual gravitation of the parts of the Earth. If the energy is taken out of the rotational energy of the Earth, we need again a mechanism of non-linear acceleration, and by arguments similar to those above, it follows that the effect is insufficient. Hence, in any event the equinoctial precession cannot be an adequate cause of the observed motions of the core.

Another phenomenon to be considered is the gradual slowing down of the Earth's rotation as astronomically observed and attributed to the fraction engendered by the lunar tide in the oceans. The problem is discussed in detail by JEFFREYS [1929]. Tidal friction would be entirely negligible in a homogeneous ocean and the actual frictional effects occur where the tidal flow is channelled into shallow basins, leading to a tremendous enhancement of the frictional effects. There seems to be no analogue to this in the core, and in our previous estimate of the tidal accelerations in the core we implicitly assumed that the fluid formed a homogeneous layer of some depth. We may conclude that the tidal phenomena in the core do not appreciably contribute to the deceleration of the Earth's rotation. As the mantle is decelerated, the core must also be decelerated at the same average rate. This leads to a slight difference in angular velocity between mantle and core which then engenders a frictional torque whereby angular momentum is transmitted from the core to the mantle. By a very simple and cogent argument which we shall not repeat here, BULLARD [1948, 1949] has shown that this process cannot give rise to any appreciable internal motions of the core.

There is one other perturbation of the central body which we might consider. The Earth describes a monthly orbit about the common center of gravity of the system Earth-Moon. The centrifugal force of this motion is opposed by the gravitational attraction of the Moon. Since the mantle and the central body have somewhat different ellipticities, their equilibrium distances from the center of the orbit differ slightly from each other. If this effect is calculated, it is found to be of a negligible magnitude.

While a quantitative calculation of all lunar influences on the core would be a very difficult matter, the preceding estimates enable us to say with some confidence that all these effects are very small compared to those derived from the convective models.

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PROGRESS REPORT, SEISMOLOGICAL LABORATORY, CALIFORNIA INSTITUTE OF TECHNOLOGY, 1949

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(Contribution No. 532, Division of the Geological Sciences,
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In addition to those published during 1949 (list at end) the following research activities have been carried forward.

Instrumental development under Benioff has included an experimental model seismometer using a capacity transducer. The pendulum is of the torsion type with an inertia reactor of approximately one kg mass. Damping is provided by a coil moving in a magnetic field with leads attached to the upper and lower torsion ribbons which are insulated from each other. In this way damping can be adjusted with an external resistance connected to the coil and any change in period brought about by reaction of the magnetic field on the coil is accounted for. The variable capacity consists of a fixed plate separated about one millimeter from a movable plate attached to a light boom on the cylinder. The mechanical magnification is approximately five. The seismometer capacitor serves to couple the grid and plate circuits of a tuned-grid tuned-plate oscillator. For small movements of the pendulum the amplitude of oscillation varies very nearly linearly with the amplitude of the pendulum displacement. A voltage output of seismic frequency is derived from the oscillator grid leak and is coupled to a one-stage amplifier by means of a resistance-capacity network. The output of the amplifier is also coupled through a resistance-capacity network. The time constant of the coupling circuits are such as to respond to seismic waves up to 60 sec-period. A short-period galvanometer (0.1 to 0.3 sec) is used for recording.

Another form of capacity transducer instrument is also being developed. In this instrument the seismometer condenser forms the capacity element in a tuned circuit of an electron coupled oscillator so that movement of the pendulum varies the frequency of the oscillator. The plate circuit of the electron coupled oscillator is coupled to a frequency modulation discriminator and the output of the discriminator is coupled through a capacity-resistance network to a recording galvanometer.

At the suggestion of R. Gilman, satisfactory compensation of the thermal drift of the spring of the Benioff variable reluctance seismometer has been accomplished by the use of lucite spacers in the supporting columns or a lucite cylinder within the steady mass. In either case the length of the lucite member is so chosen that its greater expansion as compared to steel offsets the thermal change in length of the loaded spring.

Progress has been made on the microseism project sponsored by the Geophysical Research Directorate of the Cambridge Field Station, AMC, U. S. Air Force. Two horizontal-component moving-coil electromagnetic instruments with periods of 2.0 sec have been completed. A preliminary model recorder having two galvanometers with periods of ten sec each is in operation. The optical system is so arranged that a single recording beam is reflected from both galvanometer mirrors. Additional fixed mirrors effect a 90° rotation of the plane of vibration of the light spot reflected by the first galvanometer. The movement of the light spot is thus a magnified horizontal projection of the ground motion. Recording is accomplished on 35 mm film exposed at rest for 50 sec every five minutes for a few hours every week day. Time marks are provided by interrupting the recording beam with a rotating shutter once per second. Figure 1 is a copy of (a) short period microseisms during a local rain storm; (b) ordinary 6 ± sec-period microseisms; and (c) surface waves from a teleseism.

Studies on earthquakes and rock creep by Benioff have been continued. In aftershock sequences it has been observed that energy stored in the creep mechanism in the fault rock is largely released as heat and may vary from approximately one-half to two times the energy released in the principal earthquake in the form of seismic waves. In sequences having both compressional and shear aftershock activity the delay in onset of the shear phase is presumably explained on the

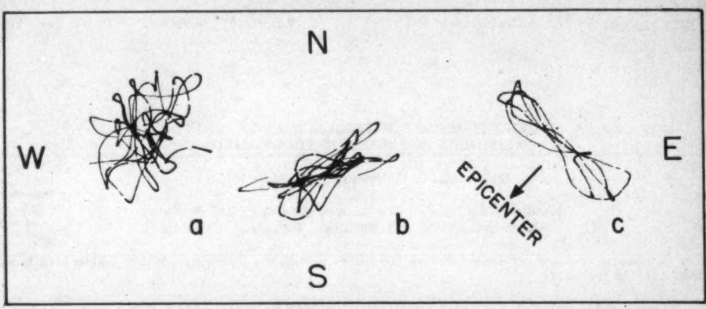

Fig. 1--Records written by Benioff vector recorder; (a) short-period microseisms superposed upon long-period microseisms; (b) long-period microseisms; (c) surface waves of earthquake, December 29, 1949

basis of the anomalous volume change in rock discovered recently by BRIDGMAN [1948].

Benioff has concluded from investigation of earthquake sequences in the Tonga Kermadec and South American region that oceanic deeps are presumably produced by the downwarping of great faults which dip under the continents. It is suggested by him that the origin of these great faults is a result of the differential density between the heavier oceanic rocks and the lighter continental rocks. The horizontal differential stress resulting from the density difference produced the sloping fault at the contact between the two rock masses. The continued activity on the fault may be attributed to a secular increase in temperature of the Earth, especially if there is an increasing temperature differential between the oceanic and the continental rock.

The accumulated strain-rebound curve of all world earthquakes of magnitudes 8.0 to 8.6 which have occurred since 1904 exhibits a saw tooth shape with very nearly linear segments. The serrations decrease regularly with time in amplitude as well as in period. If a proposed interpretation is correct, the curve indicates the following conclusions. (1) World earthquakes in this magnitude range are not independent events. They are related in some form of world-wide stress system. (2) From 1908 to 1950 the total secular strain accumulated at a remarkably constant rate. (3) This strain was released in five active periods of decreasing lengths separated by quiescent intervals of very small or no activity. (4) During the active periods the strain release proceeded at approximately twice the rate of the secular strain accumulation.

A study of the Mount Pelée sequence of volcanic eruptions of 1902 as reported by Jaggar indicates that this sequence may have been produced by compressional elastic creep of the volcano rock in much the same manner as the aftershock sequences of earthquakes are produced. We may assume that the rock was in a state of compression preceding the initial eruption and that the first outbreak was produced by elastic rebound of the rock. The vent then became sealed by a liquid column or otherwise and the volcano remained relatively quiescent until the elastic afterworking of the rock raised the pressure sufficiently to break the seal and thus generate the second outbreak. The process was repeated seven times more at lengthening intervals in accordance with the creep characteristics of the rock.

On August 6, 15^h30^m PST about 156,000 lb of explosives (DuPont Nitramon) were fired at a quarry of the Minnesota Mining and Manufacturing Company, near Corona, Southern California, (latitude 33° 50!81 N, longitude 117° 30!36 W) to loosen rock for commercial purposes. Careful preparations were made for the timing and recording of this blast. A party of staff members of the Department of Terrestrial Magnetism of the Carnegie Institution of Washington, headed by M. Tuve, participated in the arrangements. The late W. J. Rooney arranged the firing of the blast to coincide with a radio time signal. A portable seismograph of the Seismological Laboratory recorded the explosion at a distance of about 300 m from the shot point, thus providing an independent determination of the origin time. The eight permanent stations and four temporary stations of the Seismological Laboratory recorded the P waves produced by this explosion. In addition, seismograms recorded by short-period Benioff verticals were kindly furnished from the stations at Boulder City, Pierce Ferry, and Overton, by Frank H. Werner, Chief of the Lake Mead Seismological Survey (United States Coast and Geodetic Survey). Data from Tuve's records are not available. Preliminary investigation by B. Gutenberg indicates that the first P-wave at the near stations was propagated with a mean velocity of about 5.7 km/sec. Arrival times beyond 50 km show that the velocity increases rapidly (possibly discontinuously) at a depth of about five kilometers to about 6.5 km/sec. These findings are in agreement with travel times observed in previous blast records which were discussed by Wood and Richter. Pn is clearly recorded at Haiwee (Δ = 257 km) and Pierce Ferry (Δ = 407 km) and probably also at Santa Barbara, Boulder and Tinemaha. The resulting velocity is 8.2 km/sec. No phases have been found thus far which definitely establish a velocity for longitudinal waves at depths between about 12 km and the Mohorovičić discontinuity. Consequently the depth of the Mohorovičić discontinuity cannot be calculated exactly from these data. However, it cannot be very far from 34 km in this region.

The fact that in Southern California, as well as in other regions of the world, appreciably higher velocities of longitudinal waves in the uppermost layers are found from records of surface explosions than from earthquake records, leads to the conclusion that the accepted interpretation of phases in earthquake records is not entirely correct. The finding by several seismologists that in earthquakes the transverse waves seem to start earlier than the longitudinal waves is an additional indication for incorrect interpretation of the records. By use of seismograms investigated previously in records of Southern California earthquakes Gutenberg concluded that the wave called $\bar{P}$ for distances of less than about 70 km, is not connected with the phase indicated by the same symbol for greater distances. He suggested that the direct wave from the source to the station is given by $\bar{P}$ at distances up to about 70 km and from there on by Py; travel times as well as amplitudes of these two phases plotted as a function of distance merge without discontinuity. For the phases $\bar{P}$ and Py at greater distances new interpretations must be found. Gutenberg suggested tentatively that one or both may be caused by a low velocity layer at a depth of about 15 km. Such a low velocity layer is to be expected if an appreciable percentage of quartz is present at this depth, since the elastic constants of quartz decrease noticeably with temperature as the transition point from alpha- into beta-quartz is approached. From laboratory experiments this point should occur at a depth of roughly 25 km. Amplitude data, published previously, indicate that the amplitudes of the direct P wave decrease faster with distance than should be expected in case of a velocity increasing with depth in the region of the focal point or even if the velocity does not change with depth in this layer.

A complete reinterpretation of the seismograms of shocks in Southern California (and similarly in other regions) is consequently necessary. Since the installation of the new temporary stations, a number of small earthquakes have been clearly recorded at six to eight stations within 160 km. From the readings of seven such shocks during 1949, C. F. Richter has derived a mean velocity for P of 6.3 km/sec.

In a reinterpretation of records from Southern California earthquakes (1933 to 1941) Gutenberg found it sufficient to use the same locations for epicenters he had found previously. He plotted all observations of S-P for distances up to 120 km as a function of epicentral distance. On the assumption that the average velocities V of longitudinal and v of transverse waves are constant in the layers between the source and the surface, he calculated the difference $C = 1/v - 1/V$ and the average focal depth without making any assumption as to the origin time; he found that $C = 0.115$ and that the focal depth for most of these shocks is about 16 km (previously 18 km). He then used observed ratios of time differences, for the S-waves at two stations for a given earthquake to the corresponding time difference for P to find the ratio of the two velocities. Combining the resulting value of 1.73 with $C = 0.115$, he calculated that the mean velocities between the source and the surface are tentatively 6.35 km/sec for the longitudinal waves and 3.67 km/sec for the transverse waves. Both figures correspond very well to the findings from artificial explosions. The ratio of P-O to S-P is then 1.37. Using this value it is now possible to calculate the origin time of those shocks for which time intervals S-P are available. In most instances it was found that the origin time was one second or more later than had been found previously. Since the Pasadena travel time curves are based on the origin time of nearby shocks, all these travel times must now be decreased by about one second or slightly more.

H. M. Mooney has completed his thesis research on the amplitude ratio pP/P as a function of epicentral distance and focal depth. He finds that this ratio is increasingly too small relative to the theoretical expectation as the focal depth increases.

The amplitudes, periods, and travel times of P′ and P″ have been investigated by M. E. Denson. Results indicate that the epicentral distance of the main P′ focal point varies with period: for longer period waves the focal point occurs near 143 degrees; for shorter period waves near 147 degrees. The position of the end of the reversed segment of the travel time curve for waves through the outer core is believed to be near 159 degrees. Travel times show that the P′ and P″ waves are recorded as three groups separated by intervals of ten and 20 sec. Each group has its corresponding focal point. Period differences observed in the waves of the three groups indicate that the core waves suffer dispersion. From 110 degrees to greater distances the period of the P″ phase increases with increasing distance; near 110 degrees short period waves carry the largest fraction of the energy. This relation may be interpreted as a consequence of rapid velocity increase with increasing depth at the transition to the inner core.

K. Ergin has investigated the energy partition and especially zero points and extreme points of derived reflected and refracted waves resulting from a ray incident at either side of a rock-water boundary at a variable angle. He found that (1) Poisson's ratio of the solid medium is a dominant factor effecting the functional behavior, zero points and extreme points of the reflected and refracted waves in the solid. (2) Changes in the ratio of the two longitudinal wave velocities chiefly affect the refracted wave energy. (3) The effect of change in the density ratio is very slight. (4) Peculiar effects occur in the immediate vicinity of the critical angle of incidence. (5) At a critical angle of incidence, all incident energy goes into a reflected wave of the same kind as the incident wave. (6) Most of the incident energy is reflected either as P or as S wave depending upon the angle of incidence.

Ergin also is extending the study of the displacement ratios of PcP, ScP, PcS and ScS to P and S initiated by MARTNER [1950] to intermediate and deep focus earthquake seismograms. The results obtained so far indicate that the observed ratios of the horizontal displacements of the waves reflected as P waves at the core boundary to that of the incident wave (that is, PcP/P and ScP/SV) are five to ten times larger than the corresponding ratios calculated from travel time curves. The observed ratios of the vertical displacements are only two or three times larger than the theoretical ones. For those waves reflected as S waves at the core boundary, the observed ratios (that is, ScS/S, PcS/P) of the horizontal displacements are in fairly good agreement with the theoretical ratios, but the observed ratios of the vertical components are considerably larger than the theoretical ratios. No appreciable effect of focal depth on the discrepancies is indicated.

P. Dehlinger has investigated the direction of transverse waves from numerous small local earthquakes in Southern California. The SH components of these waves demonstrate strikingly consistent directions of the first motion within the area, while the SV components appear to be less consistant. The first motion of SH has a component in the same sense as the fault block displacements. The data suggest that two fault types are prevalent in the area. The predominant one is of the San Andreas type, a northwesterly trending righthand transcurrent fault. The other may involve thrust faulting. The two types seem to occur separately within limited areas.

The Laboratory staff have contributed to the revision of the book, "Internal Constitution of the Earth," sponsored by the National Research Council (B. Gutenberg, editor).

Three additional vertical Benioff seismographs with short-period galvanometers have been completed. Instruments now available for temporary stations have recorded as shown in Table 1. We are indebted to the officers and personnel of the Desert Hot Springs Chamber of Commerce, Pomona College, Perris Grammar School, Club San Mortiz (Crestline) and the Southern California Telephone Company (El Cajón) for their cooperation.

Table 1--Temporary stations

Station	Latitude N	Longitude W	Height	Dates of operation
	° ′	° ′	m	
Desert Hot Springs	33 57.7	116 30.0	330	Dec. 10, 1948-Mar. 24, 1949
Pomona	34 05.9	117 42.6	350	Feb. 2, 1949-date
Perris	33 46.8	117 14.0	440	Apr. 28, 1949-date
Crestline	34 14.6	117 15.7	1400	May 19, 1949-date
Chine Lake	35 49.0	117 35.8	766	July 8, 1949-date
El Cajón	32 47.6	116 57.3	135	July 26, 1949-Aug. 16, 1949

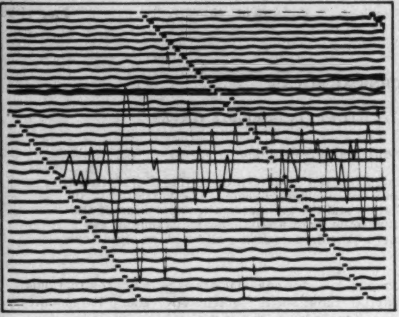

Fig. 2--Record of earthquake which was accompanied by damage to oil wells near Long Beach on November 17, 1949; Wood-Anderson torsion seismometer, T = 6 sec, at Pasadena; distance about 45 km

An earthquake on November 17, 1949, damaged over 200 oil wells on and near Terminal Island (Long Beach Harbor), largely by displacement at depths near 1700 ft. It was reported felt by very few persons. Large seismograms were written at Pasadena (see Fig. 2); the shock was recorded at nearly all the Southern California stations, as well as at Boulder City and Tucson. The main disturbance began with small motion. Plotting the recorded times against distance from Terminal Island gives an alignment consistent with an epicenter near there. The estimated origin time is $17^h 19^m 52.4^s$ PST. The seismograms differ from those of ordinary earthquakes, normal phases cannot be identified, and there is a train of large late long-period waves, presumably surface waves, as might be expected from a shallow source. Occurrences of the same type, with similar seismograms, but with smaller amplitudes were observed in 1933 and 1947.

The largest earthquake during 1949 (magnitude about eight) occurred on August 22, 4^h off the Queen Charlotte Islands (British Columbia). Routine bulletins for teleseisms have been issued through March, 1949 and for local shocks through June, 1949 under supervision of C. F. Richter.

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TECHNICAL NOTE

EFFECT OF A PARTIAL CUTOFF ON SEEPAGE RATES

John S. McNown and En-Yun Hsu

Abstract--Conformal transformations are used in determining the effectiveness of a partial impervious barrier in an idealized seepage channel. The variation of the quantity of flow with the extent of penetration and the geometrical proportions of the channel is obtained in terms of elliptic functions; both analytical and graphical results are given. Only for effectively short channels do cutoff walls reduce significantly the rate of flow.

A theoretical solution for an idealized case of two-dimensional seepage indicates that only for effectively short seepage channels will partial cutoff walls significantly reduce the quantity of flow through porous strata. The ratio of the discharges with and without the cutoff is a function of the extent of the impervious barrier and the length of the channel relative to the stratum thickness. This relation can be expressed in functional form

$$q/q_o = f(d/h\ ,\ L/h)$$

in which the various dimensions are defined in the diagrammatic sketch (see Fig. 1). Because of the analogy between seepage and irrotational flow, the methods of conformal mapping are applicable in the evaluation of the desired functional relationship. If the coefficient of permeability is greater for horizontal than for vertical flow, as for stratified media, the effective length is reduced proportionately.

In a field installation in which a reduction of seepage losses was desired, the pervious channel was composed of two strata of nearly equal permeability, the upper a sand and gravel deposit and the lower a porous sandstone. Because it was practicable to place an impervious barrier composed of sheet piling only in the upper layer, determination of the effectiveness of such a procedure was necessary. A uniformly thick stratum of finite length with a central partial barrier and the flow conditions shown in Figure 1 was assumed. A related problem has been solved by Muskat, in which the effect of a partial cutoff on the seepage in an infinitely long pervious stratum beneath an impervious dam was determined [MUSKAT, 1937]. However, significant differences between the two sets of boundary conditions, in particular the finite length of the pervious stratum, made it necessary to conduct an analysis of this specific problem. The principal steps leading to the solution are presented in the following paragraphs.

Determination of the unknown function in (1) by the methods of conformal mapping depends upon the evaluation of the relationship between the desired flow pattern shown in Figure 2a (inverted for mathematical simplicity) and the simple rectangular pattern of streamlines and lines of constant pressure in Figure 2b. Since the pattern is two-dimensional, these methods of elementary complex function theory can be used if the velocity is derivable from a potential and if the flow is continuous. The first condition is satisfied if the well known Darcy law is applicable, that is, if

$$v_s = k\,(\partial p/\partial s) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

defines the velocity component in any arbitrary direction the quantity kp is then the velocity potential. For satisfaction of the condition of continuity, the medium must be homogeneous so that k is a constant, and no irregularities such as air pockets may exist, so that the specified boundaries truly bound the flow. These conditions, which are satisfied for a variety of ground-water flows, have been discussed in detail by MUSKAT [1937], JACOB [pending], and others. In combination, the criteria lead to the Laplacian equation which is encountered in the

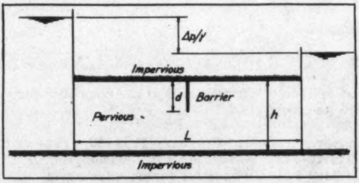

Fig. 1--Definition sketch for idealized flow conditions

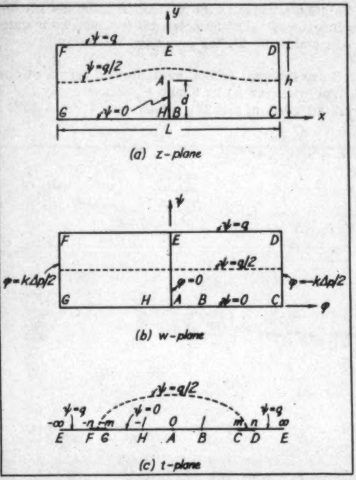

Fig. 2--Conformal transformation planes: (a) flow pattern; (b) complex potential; (c) auxiliary plane

solution of physical problems in many different fields.

Transformation of an irregular flow pattern like that in Figure 2a to one of simpler form is greatly expedited by the Schwartz-Christofel theorem which provides a direct method for transforming the interior of any polygon in one plane, for example, the z-plane, into the upper half of another, for example, the t-plane, [STREETER, 1948]. For mathematical convenience, each plane is considered to be complex with real and imaginary axes, and the transformed polygon becomes the real axis in the auxiliary plane. The theorem can be written

$$\frac{dz}{dt} = \frac{M}{(t-a_1)^{\alpha_1/\pi}(t-a_2)^{\alpha_2/\pi}(t-a_3)^{\alpha_3/\pi}(t-\cdots} \quad . \; . \; (2)$$

in which M is a constant of proportionality, α_1 is the exterior angle at one of the vertices of the polygon in the plane and a_1 is the corresponding point on the real axis in the transformed plane. As many factors appear in the right-hand denominator as there are vertices in the finite portion of the z-plane. Since (2) is a differential equation, its definition provides a useful relationship only if the indicated integration can be performed.

The polygonal boundaries in Figure 2ab can both be transformed into the straight line in Figure 2c with a one-to-one correspondence of the three patterns; analogous points in the three planes are designated by the same letter. Each of the diagrams can be thought of as a flow pattern, and the bounding streamlines are indicated in each. In the t-plane, flow results from a line source of variable strength FG and a symmetrical line sink CD. The flow pattern in the lower half of the t-plane is identical to that in the upper half, but has no effect on the desired relationships. An intermediate streamline is indicated schematically in each plane for illustrative purposes.

Since each of the α-values in Figure 2a is $\pi/2$, except for the $-\pi$ value at A, the seven factors in the transformation equation relating t and z can be written in the form

$$Z_2 - Z_1 = M\int_{t_1}^{t_2} \frac{t\,dt}{\sqrt{(t^2-n^2)(t^2-m^2)(t^2-1)}} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (3)$$

inasmuch as three of the a-values can be arbitrarily assigned (0, +1, -1), and the remaining four ($\pm$m, $\pm$n) must be evaluated otherwise. In a similar way, w can be expressed as a function of t,

$$w_2 - w_1 = N\int_{t_1}^{t_2} \frac{dt}{\sqrt{(t^2-n^2)(t^2-m^2)}} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (4)$$

the values $\pm m$ and $\pm n$ are again used to assure correspondence between the w-plane and the z-plane. The determination of the necessary relationships between the various parameters can be accomplished by evaluating these definite integrals between pairs of points for which corresponding values occur in the various planes, $\sqrt{-1}$ being indicated by the symbol i:

At A	z = id	t = 0	w = 0
B	z = 0	t = 1	(unnecessary)
C	z = L/2	t = m	w = -kΔp/2
D	z = ih + L/2	t = n	w = qi - kΔp/2

The discharge between two streamlines per unit distance normal to the x-y plane is by definition equal to the difference of the values of the stream function ψ, and the potential is taken as negative at C and D so that flow will be both from left to right and from higher pressure to lower.

If the substitution $\zeta = t^2$ is made in (3), the definite integral can be evaluated in terms of complete and incomplete elliptic functions of the first type, designated as K and F, respectively [PIERCE, 1929]. In this way d, L, and h can be related to m, n, and M. Integrating from A to B

$$d = -\frac{M}{2i}\int_0^1 \frac{d\zeta}{\sqrt{(\zeta-n^2)(\zeta-m^2)(\zeta-1)}}$$

$$= \frac{M}{2}\int_0^1 \frac{d\zeta}{\sqrt{(n^2-\zeta)(m^2-\zeta)(1-\zeta)}} = \frac{M}{\sqrt{n^2-1}}\,F\left(\frac{1}{m},\sqrt{\frac{n^2-m^2}{n^2-1}}\right)$$

Similarly, from B to C

$$L = M\int_1^{m^2} \frac{d\zeta}{\sqrt{(n^2-\zeta)(m^2-\zeta)(\zeta-1)}} = \frac{2M}{\sqrt{n^2-1}}\,K\left(\sqrt{\frac{m^2-1}{n^2-1}}\right)$$

Finally, from C to D

$$h = \frac{M}{2}\int_{m^2}^{n^2} \frac{d\zeta}{\sqrt{(n^2-\zeta)(\zeta-m^2)(\zeta-1)}} = \frac{M}{\sqrt{n^2-1}}\,K\left(\sqrt{\frac{n^2-m^2}{n^2-1}}\right)$$

Introduction of Δp and q as functions of m, n, and N can be accomplished through applications of (4) in a similar manner. Integrating from A to C

$$\frac{-k\Delta p}{2} = N\int_0^m \frac{dt}{\sqrt{(n^2-t^2)(m^2-t^2)}} = \frac{N}{n}\,K\left(\frac{m}{n}\right)$$

and from C to D

$$q = N\int_m^n \frac{dt}{\sqrt{(n^2-t^2)(t^2-m^2)}} = \frac{N}{n}\,K\left(\sqrt{1-\frac{m^2}{n^2}}\right)$$

The discharge without a cutoff q_0 is equal to $-k\Delta p\,h/L$.

From the foregoing equations a relationship of the desired type is obtained in parametric form, the quantities M, N, and Δp having been eliminated by taking ratios

$$\frac{d}{h} = \frac{F\left(\frac{1}{m},\sqrt{\frac{n^2-m^2}{n^2-1}}\right)}{K\left(\sqrt{\frac{n^2-m^2}{n^2-1}}\right)} \quad \text{(5)}$$

$$\frac{L}{h} = \frac{2K\left(\sqrt{\frac{m^2-1}{n^2-1}}\right)}{K\left(\sqrt{\frac{n^2-m^2}{n^2-1}}\right)} \quad \text{(6)}$$

$$\frac{q}{q_0} = \frac{K\left(\sqrt{1-\frac{m^2}{n^2}}\right)}{2K\left(\frac{m}{n}\right)} \frac{L}{h} \quad \cdots\cdots\cdots\cdots \quad (7)$$

If numerical values are assigned to m and n satisfying the inequality n > m > 1, corresponding simultaneous values may be obtained for d/h, L/h, and q/q$_0$. Since this process will lead to a tedious trial-and-error solution for a given problem, the plot in Figure 3 was prepared to make possible the immediate determination of an approximate result throughout the practicable range of values. For large values of d/h and for L/h less than one or greater than six, the necessary values of m and n were very nearly equal, such pairs as (1.00001, 1.01) and (14.99999, 15.0) being used. The values of the corresponding angles used in evaluating the elliptic functions (angles having sines equal to the expressions in parentheses in (5), (6), and (7)) approach 90° so closely that difficulties are encountered because of limitations in the best tables available [LEGENDRE, 1826]. For those portions of the curves which could not be computed readily the trend is indicated by broken lines.

The limiting values for L/h of 0 and ∞ can be rationalized to verify the limiting values of the functional result. For a very short channel the quantity of flow varies directly with the open area or inversely with the penetration as shown by the diagonal straight line. For an infinitely long

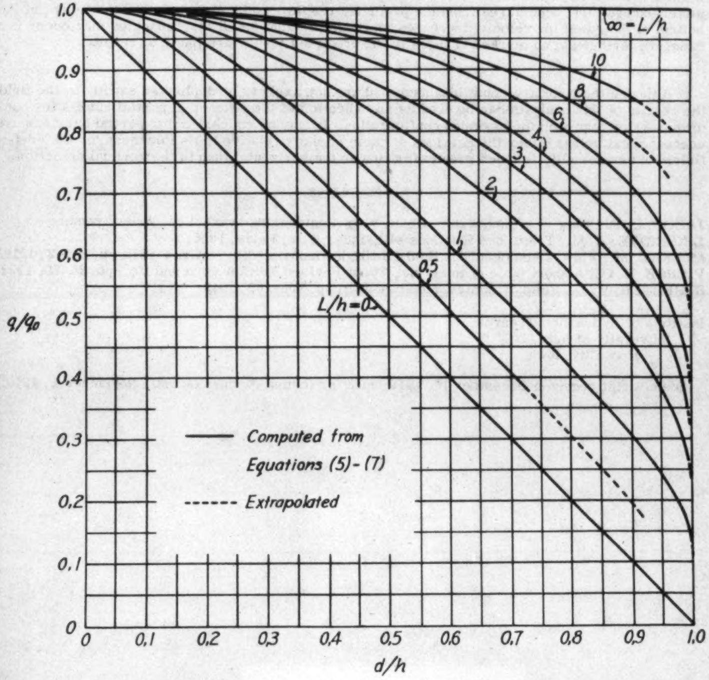

Fig. 3--Effect of a partial cutoff on the rate of seepage

channel the additional resistance caused by a barrier is negligible, except for a complete cutoff, so that the upper margin and lower right-hand corner of the diagram depict this limiting case. A notable feature of the result is the tendency for the curves to approach the condition of infinite length for relative lengths which are not great. For example, in the problem which led to this analysis the penetration ratio was 1/2 and L/h was ten. For these values the reduction in quantity of flow is only 3.5 per cent.

Unequal permeabilities in the vertical and horizontal directions result in a type of inhomogeneity often encountered, but one which does not preclude analysis. In fact, for a given ratio k_x/k_y of the two permeability coefficients, Figure 3, can be applied directly if an effective length ratio L'/h is computed from the simple relationship

$$L'/h = (L/h) \cdot (k_y/k_x)$$

If k_x is considerably larger than k_y, and values of k_x/k_y in the range between 10 and 50 sometimes occur, the effective length ratio L'/h is proportionately less than the geometric length ratio L/h and the effectiveness of the cutoff is correspondingly greater. In the illustration cited, if k_x/k_y were equal to five, the effective length ratio would be two. The corresponding reduction in discharge is 18 per cent (from Fig. 3). as compared to 3.5 per cent for equal permeability coefficients.

The foregoing solution, which is based on Darcy's law, is applicable only if the flow is truly laminar. If the Reynolds Number of the flow based on the mean velocity and mean particle diameter exceeds ten, departures from the predicted results can be expected. Furthermore, at the bottom of the cutoff the velocity increases markedly, so that small discrepancies may occur in any case. Nevertheless, no marked changes in the predicted trends are likely to result.

Although the boundary conditions assumed are not likely to be duplicated exactly in the field, the results of this analysis can be used as an indication of the efficacy of partial cutoffs for conditions which approach the assumed configuration. It may be concluded that partial barriers are uneconomical unless either the pervious stratum is short relative to its thickness, or the coefficient of permeability is much greater for flow in the horizontal than in the vertical direction.

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DISCUSSION OF "THE RELATION OF GEOLOGY TO DRY WEATHER STREAM FLOW IN OHIO" by WILLIAM P. CROSS

[Trans., v. 30, pp. 563-566, 1949]

Raphael G. Kazmann (Ranny Method Water Supplies, Inc., 841 Alton Ave., Columbus 3, Ohio)--The concepts presented in this paper are of great value to the ground-water hydrologist. They indicate one way to analyze and conveniently use the vast accumulation of stream-flow records collected by governmental agencies as a tool for geophysical reconnaisance. To be usable as a method of geophysical prospecting for ground water, stream-flow records must first be transformed into flow-duration curves. Then, by means of the 90 per cent index, chosen by Cross, or other, more suitable indices, the ground-water storage characteristics of the surficial deposits of drainage basins can readily be determined. Where there is good indication of extensive ground-water storage, it is probable that large supplies of ground water can be developed for use.

Even more important is the paper's inference that isolated measurements of dry-weather flow can yield significant and important hydrogeological information. For if the dry-weather flow in an otherwise ungaged stream is compared, on a unit area basis, with a simultaneous measurement on, and with the long-term flow-duration curves from a nearby watershed, valid inferences can be made as to the water-bearing capabilities of the surficial formations. Thus, over a period of a few years, by means of one or two isolated measurements at points where continuous stream-flow records are not available, the hydrogeology of limited areas of interest can be determined inexpensively. Intensive exploration can then be confined to the most promising areas, thus saving time and money. This particular technique was used by the writer in studying the probable yield of a new ground-water supply for Canton, Ohio [KAZMANN, 1949]. The making of simultaneous measurements of Sandy Creek and West Branch of Nimishillen Creek enabled the construction of a synthetic flow-duration curve for the latter using the Sandy Creek flow-duration curve as a basis. A sound estimate of the water available for infiltration to the aquifer was then made and engineering computations, which have since proved accurate, predicted the firm quantity of ground water available.

The Cross technique is undoubtedly applicable to any area where artificial regulation of stream flow is not important. However, whereas the 0.1 cfs/sq mi for 90 per cent of the time can be used to differentiate promising surficial deposits from unpromising ones in Ohio, climatological differences will change this criterion when the technique is applied to other areas. In Mississippi or Louisiana, where the annual rainfall is much higher, and differently distributed, a higher figure might have to be used. Geographical differences would also tend to modify the Cross criterion, as in Maine, for example, where the many lakes regulate the dry-weather flow of streams. As with all other hydrologic data, the mere preparation of flow-duration curves will not eliminate the need for skilled and experienced hydrologists. But, using such curves in the manner described by Cross, the hydrologist can do a much better job and cover a much alrger area than could previously be done with the same expenditure of time and effort.

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KAZMANN, RAPHAEL G., The utilization of induced stream infiltration and natural aquifer storage at Canton, Ohio, Econ. Geol., v. 44, no. 6, pp. 514-524, September-October, 1949.

William P. Cross (U. S. Geological Survey, Columbus, Ohio, Author's closure)--The technique described by Kazmann for applying dry-weather flow data to ground water problems has been used successfully in Ohio and elsewhere. Generally, high dry-weather stream flow in indicative of large storage in ground-water bodies. However, low dry-weather stream flow does not necessarily mean a lack of possible ground-water supplies. Comparatively large ground-water aquifers may be present at such depths that they have little or no influence on stream flow.

The discussion by Wilson of the effect of climatological factors on flow-duration curves deserves some additional explanation. It was the author's intention to show that the shapes of the lower ends of duration curves based on long records (15 yr or more) reflect geologic effects. This appears to be true in Ohio, where the average meteorologic factors are fairly homogeneous, with little difference in average rainfall, with slight, if any, orographic effect, and with the average rainfall distributed evenly throughout the year. Comparisons of dry-weather flow indices based on short records, (unless adjusted by correlation with nearby long-term records), or for areas with substantial differences in rainfall characteristics, might be very misleading.

Since the publication of this article, a report on flow-duration characteristics of Ohio streams has been published [CROSS, 1949].

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DISCUSSION OF
"A MANOMETRIC SETTLING VELOCITY TUBE"
by MARTIN A. MASON

[Trans., v. 30, pp. 533-538, 1949]

Karl S. Pister (University of Illinois, Urbana, Illinois)--This report presents a very interesting solution to the problem of instrumentation in determining the settling characteristics of particles in the sand-size range. In the introductory paragraph of the paper the author states the principal characteristics of particles upon which settling velocity depends. For the particular technique described here it seems that certain other factors (as mentioned briefly by the author) should be considered.

Of prime importance is the degree of mutual interference among individual particles during the settling period. If the concentration of particles per unit volume of suspending fluid is too large, segregation of particles according to size will not take place in any reasonable length of settling tube. According to DALLAVALLE [1948] a concentration of particles of from two to five per cent gives satisfactory results. No specific information is given in this paper for comparison with this figure, although its significance should be considered.

In addition the preparation of the sample prior to making an analysis is important. Ordinarily the settling characteristics of particles in a saturated state are desired. Depending upon the porosity and surface texture of individual particles of a sample the time necessary for complete saturation is variable. A particle may exhibit markedly different settling characteristics in a partially saturated state than when completely saturated. As an extreme example the writer is familiar with a shale that floats when air-dry but settles rapidly after becoming saturated. Thus, it is of considerable importance that all samples be thoroughly saturated prior to making an analysis. It is suggested that a Lucite cylinder would prove useful in place of the steel pipe to permit actual observation of particle behavior, interference and segregation, during a particular test.

From the standpoint of theory it would be interesting to attempt to correlate the results of this type of analysis with those of an ordinary sieve analysis to determine particle size-frequency distribution. Assuming uniform shape characteristics for all particle sizes (reasonably correct for particles of similar origin), particles of equal size would settle with the same velocity. It should then be possible to use appropriate equations to calculate for various size ranges their respective settling velocities. From this information a settling velocity distribution curve could be synthesized and compared with the actual observed one. The manner of calculation of the settling velocities is dependent upon particle size. Small particles follow the law of viscous resistance while larger ones are governed by turbulent resistance. For the intermediate size range a transitional form of resistance occurs. For quartz particles in water is it considered approximately that particles less than 85 microns follow Stoke's Law and those larger than 2000 microns follow the turbulent laws of resistance. Samples in the sand-size range would, quite possibly then, contain particles that each of the three types of equations of motion would govern. Such formulae can be found in DALLAVALLE [1948]

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NOTES AND PERSONALIA

NOTES

Colloquium on Plastic Flow and Deformation within the Earth--A Colloquium on Plastic Flow and Deformation within the Earth, sponsored jointly by the International Union of Geodesy and Geophysics, and the International Union of Theoretical and Applied Mechanics, is to be held in Washington, D. C., September 12-14, 1950. It is anticipated that this will be a small group, with about 12 from abroad and a similar number from the United States. L. H. Adams, Director of the Geophysical Laboratory, Carnegie Institution of Washington, 2801 Upton Street, N. W., Washington, D. C., is Secretary of the Joint Organizing Commission.

Johns Hopkins field course in climatology--The summer field course of the Laboratory of Climatology of Johns Hopkins University, which is at Seabrook, New Jersey, will extend from June 1 to October 1, 1950. Rudolf Geiger, the world's leading microclimatologist, will be in residence, advising on current research and giving this course. In addition to the five regularly enrolled graduate students, there will be two students from Portugal and several Air Force officers in attendance.

New geophysical journal in India--Copies of volume 1, number 1 of the Indian Journal of Meteorology and Geophysics have arrived in the United States. The Journal, issued quarterly, is published by the India Meteorological Department in Delhi under the direction of an editorial committee consisting of S. K. Banerji, B. L. Gulatee, S. K. Pramanik, F. Ermisch, L. A. Ramdas, R. D. Dhir, S. Mull, and S. K. Das. The papers appearing in the first issue are listed under the bibliography in this and the next issue of the Transactions. In addition to scientific papers, the Journal carries notes relating to Indian and international scientific affairs.

Meeting of the Eastern Snow Conference--The Eastern Snow Conference held its annual meeting at Greenfield, Massachusetts, February 16-17, 1950 with about 60 in attendance. Eleven technical papers were presented. The newly elected officers are: President, Bryant L. Hopkins; Vice President, Elliot F. Childs; and Secretary, Holbert W. Fear. The next annual meeting is to be held in Lake Placid, N. Y.

Colloquium on aeolian sediments--A colloquium to discuss transport of sand in arid areas, aerodynamic problem of transport, aeolian erosion, and related meteorological and other geophysical factors is to be held in Algiers, probably in the second week of October, 1950. Plans are being made for attendance by ten French specialists and ten from other countries, and are in charge of Prof. P. Queney, Université de Paris, Faculté des Sciences, 191 rue Saint Jacques, Paris 5^{e}, France.

New geological society in Arizona--The Arizona Geological Society, composed of professional geologists and of graduate students in geology, was recently organized at Tucson. It had operated informally for about two years. The officers are as follows: President, J. H. Feth; Vice President, J. M. Hill; Secretary-Treasurer, John W. Anthony; members of the Advisory Council, B. S. Butler, L. Kenneth Wilson, A. A. Stoyanow, Roland B. Mulcahy, T. S. Lovering, and M. N. Short.

PERSONALIA

Roger Adams, Professor of Chemistry at the University of Illinois has been named Foreign Secretary of the National Academy of Sciences, succeeding Detlev W. Bronk.

Francis Birch, Professor of Geophysics at Harvard University was recently elected a member of the National Academy of Sciences.

Detlev W. Bronk, President of Johns Hopkins University, was recently elected President of the National Academy of Sciences for a four-year term beginning July 1, 1950.

Leo O. Colbert, Director of the U. S. Coast and Geodetic Survey for the last 12 years, and member of the Survey staff since 1907, retired April 7, 1950.

Ernst Cloos, Professor of Structural Geology at the Johns Hopkins University was recently elected to membership in the National Academy of Sciences.

William S. Dawson, Major in the U. S. Air Force, has been assigned to duty in the Far East on weather-communications facilities. His headquarters are in Tokyo.

Milton B. Dobrin has taken up duties as Senior Research Physicist at the Field Research Laboratories of the Magnolia Petroleum Company in Dallas, Texas.

I. G. Grossman, formerly of the U. S. Bureau of Mines, is now with the Ground-Water Branch of the U. S. Geological Survey with headquarters in Albany, N. Y.

Howard S. Rappleye, Chief, Section of Leveling, U. S. Coast and Geodetic Survey, was recently awarded the Meritorious Service Medal of the Department of Commerce.

Edward H. Smith, Rear Admiral of the U. S. Coast Guard, will retire from that service on June 30, 1950, and take up duties as the Director of the Woods Hole Oceanographic Institution.

Robert F. A. Studds, Rear Admiral, and Chief of the Division of Charts, U. S. Coast and Geodetic Survey, has been nominated by President Truman to be Director of the Survey.

Howel Williams, Chairman of the Department of Geological Sciences, University of California, was recently elected a member of the National Academy of Sciences.

C. H. M. van Bavel has taken up duties as Research Assistant Professor in the Department of Agronomy of North Carolina State College where he is conducting research in soil physics.

Harry Thomas Gisborne, Chief of the Division of Forest Fire Research at the Northern Rocky Mountain Forest and Range Experiment Station at Missoula, Montana, died suddenly on November 9, 1949 while making a survey of the Mann Gulch area where 13 fire fighters lost their lives last August. He was 56 years old. He was born in Montpelier, Vermont, and received his baccalaureate degree from the University of Michigan in 1917. He began his work with the Forest Service in 1919, and became a specialist in the measurement of forest-fire hazards, basing his method on climatological and fuel-moisture factors. He had recently been working with engineers of the General Electric Company on problems of lightning control. In 1947, he was awarded a Superior Service Medal by the Department of Agriculture. He had been a member of AGU since 1938.

Charles N. Gould, retired, of Norman, Oklahoma, died August 13, 1949, at the age of 81. He was born at Lower Salem, Ohio, and secured his baccalaureate degree from Southwestern College in 1899. He received his master's degree in 1900, his doctorate in 1906, and an honorary doctorate in 1928 from the University of Nebraska. He began his professional career as an instructor at the University of Oklahoma, occupying a chair as Professor from 1903 to 1911. From 1908 to 1911, and from 1923 to 1931, he was Director of the Oklahoma Geological Survey. In the intervening period he engaged in consulting work. From 1931 until his retirement in 1940, he was Regional Geologist for the National Park Service. He was recipient of several honors and awards, and was an authority on water resources of the Great Plains.

Henry Frank Moore, retired, of Linville Falls, North Carolina, died January 8, 1948 at the age of 80. He was born in Philadelphia, received his baccalaureate degree from Philadelphia Central High School in 1885, and his doctorate from the University of Pennsylvania in 1895. He was a naturalist with the International Fisheries Commission from 1893-1895, Chief Naturalist on the Albatross from 1896-1903, and served in the post of Deputy Commissioner of Fisheries from 1915-1923. He was the recipient of a number of honors and awards, and in World War II was a consultant regarding many of the islands of the Pacific. He had been a member of the American Geophysical Union since 1923.

Alonzo H. Tuttle, retired, District Engineer of the U. S. Geological Survey at Helena, Montana, until December 31, 1949, died March 5, 1950 at the age of 69. He was born at Richland Center, Wisconsin, attended the Teachers' College of Oshkosh in 1902-1903, and received his baccalaureate degree from the University of Wisconsin in 1910. In 1911 he became associated with the U. S. Geological Survey on surface-water studies, and since 1916 had been located at Helena. He was author and co-author of various reports and he was instrumental in the organization of snow surveying. He had been a member of AGU since 1943.

Hans Zölly, Direktor-Stellvertreter der Eidgenössischen Landestopographie in Wabern near Bern, Switzerland, died February 28, 1950, at the age of 69.

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THIRTY-FIRST ANNUAL MEETING (1950)

MINUTES, GENERAL ASSEMBLY AND SECTIONS

EVENING SESSION

The Evening Session of the Thirty-First Annual Meeting was called to order by General Secretary K. Hilding Beij at $20^{h}10^{m}$, Monday, May 1, 1950, in the Lecture Room of the National Academy of Sciences, 2101 Constitution Avenue, Washington, D. C. About 180 were in attendance. After preliminary and welcoming remarks, General Secretary Beij introduced President Bucher who delivered his Presidential Address (to be published in an early issue) on the subject "Megatectonics and Geophysics."

Following the address, General Secretary Beij relinquished the chair to President Bucher, who then introduced Merle A. Tuve who (on behalf of the Committee on the Award of the William Bowie Medal and its Chairman, Leo O. Colbert, cited Leason Heberling Adams as the Twelfth William Bowie Medalist [see pp. 341-342 of this issue]. Following this, the presentation was made by President Bucher to Dr. Adams who then responded and accepted the Medal, as set forth following the citation.

The session was then declared adjourned, and a social hour, with refreshments, followed in the Auditorium.

SMOKER

A Smoker was held in the Auditorium of the Cosmos Club Tuesday, May 2, 1950 from $20^{h}00^{m}$ to $22^{h}30^{m}$. About 170 were in attendance. The affair was informal, with no program, but offered an opportunity to renew old acquaintances and friendships and make new ones. Full advantage was taken of these opportunities, and many lively discussions ensued.

GENERAL ASSEMBLY: SPECIAL PROGRAM AND BUSINESS MEETING

The General Assembly was called to order by President Bucher at $14^{h}05^{m}$, Wednesday, May 3, 1950, in the Auditorium of the National Academy of Sciences. About 200 were in attendance. The Special Program proceeded as set forth in the Program [see p. 316].

Following this, the Business Meeting was called to order. In opening the meeting, the President pointed out that under our democratic processes, this is one of the opportunities that members have to bring their thoughts on matters pertaining to AGU to the attention of the membership. Matters, of course, may properly be brought to the attention of any officer or any Executive Committee member at any time. The minutes of the last meeting [see Trans., v. 30, pp. 460-463] were approved.

Report of the General Secretary--General Secretary Beij summarized the activities of the Union since the Thirtieth Annual Meeting. These are reported in detail in connection with the Minutes of the Meetings of the Executive Committee of October 28, 1949 [see Dec. 1949, Trans., pp. 928-939] and May 3, 1950 [to be published in the August, 1950 issue of the Transactions].

A recapitulation of the changes in membership since April 20, 1949, is as follows:

Members, April 20, 1949	4449	
Inactive to active status	1	
New Members	516	
		4966
Active to inactive	7	
Resignations	219	
Deceased	30	
Dropped	115	
		371
Membership as of April 20, 1950		4595

The Annual Report of the General Secretary was accepted.

Reports of Standing Committees

Meetings--Chairman John Putnam Marble of the Committee on Meetings announced the dates for the Thirty-Second Annual Meeting as April 30-May 2, 1951. He also reported briefly on meetings during the past year, announcing the registration of 865 at the Thirty-First Annual Meeting, with perhaps more than 1000 in attendance, and an invitation from the Western Snow Conference to join with them in a meeting at Victoria, British Columbia in the spring of 1951.

Reports of other standing committees were not presented, and will be published in the Transactions.

Reports of Special Committees

Resolutions--Chairman Earl Ingerson presented seven resolutions which were unanimously adopted, as follows:

(1) RESOLUTION OF THANKS FOR USE OF FACILITIES

WHEREAS, The success of the Thirty-First Annual Meeting of The American Geophysical Union on May 1-3, 1950, has been largely due to the ample accommodations and excellent services provided for the sessions, therefore be it

RESOLVED, That the American Geophysical Union extend its heartiest thanks to the National Academy of Sciences and to the United States Public Buildings Administration for the use of meeting rooms in their buildings and for the provision of the various services rendered in connection therewith and that the General Secretary be instructed to forward copies of this resolution to the appropriate officials.

(2) RESOLUTION OF THANKS TO THE UNIVERSITY OF CALIFORNIA

WHEREAS, The success of the Twenty-Fifth Regional Meeting at Davis, California, February 3-4, 1950, was due in large part to the space and facilities generously provided by the University of California, therefore be it

RESOLVED, That the thanks of the American Geophysical Union be extended to the University of California at Davis for the use of their meeting rooms and other facilities, and that the General Secretary be instructed to forward copies of this resolution to the appropriate authorities.

(3) RESOLUTION OF THANKS TO THE CARNEGIE INSTITUTION OF WASHINGTON

WHEREAS, The Carnegie Institution of Washington has for many years kindly furnished office, storage space, and other facilities without charge for use by the American Geophysical Union and has continued the same service for the past year, therefore be it

RESOLVED, That the American Geophysical Union express its sincere appreciation of this kind and generous action of the Carnegie Institution of Washington and instruct its General Secretary to express the thanks of the Union for the same to the President and Chairman of the Board of Trustees of the Carnegie Institution of Washington.

(4) RESOLUTION ON COMMUNICATIONS SERVICES

WHEREAS, The Secretary of Commerce in cooperation with the Foreign Service Office of the United States, the officers of the Foreign Service, and the Foreign Service Operations Staff of the Office of International Trade, have made available their special telegraphic facilities for transmitting teleseismic data to the United States Coast and Geodetic Survey, and

WHEREAS, The data have greatly accelerated and expanded the earthquake location program of the said Survey, and

WHEREAS, The data have been made available by the Survey to seismologists throughout the world to aid in the more thorough analysis of their records, therefore be it

RESOLVED, That the American Geophysical Union express its appreciation for this valuable contribution to seismological research, and be it further

RESOLVED, That copies of this resolution be sent to the Secretary of Commerce, the Foreign Service Office of the United States, the Foreign Service Operations Staff of the Office of International Trade, and the officers of the Foreign Service at the following stations: Djakorta, Java; Bombay, India; Brisbane, Australia; Cairo, Egypt; Copenhagen, Denmark; Damascus, Lebanon; Guatemala City, Guatemala; La Plata, Argentina; Lima, Peru; Malaya, Spain; Mexico City, Mexico; Perth, Australia; Rio de Janeiro, Brazil; Rome, Italy; Santiago, Chile; Strasburg, France; Sydney, Australia; Tananarive, Madagascar; and Pretoria, South Africa.

(5) RESOLUTION ON TIME SIGNAL SERVICE

WHEREAS, There is a vital need in all seismological work for accurate time service, and

WHEREAS, The time service maintained by the United States Naval Observatory, the Naval Communications Service, and the National Bureau of Standards has been of great use in this work, therefore be it

RESOLVED, That the American Geophysical Union express its appreciation of the valuable scientific cooperation and reiterate its desire for the complete continuance of this time signal service, and be it further

RESOLVED, That copies of this resolution be sent to the Secretary of the Navy, the Director of Naval Communications, the Superintendant of the Naval Observatory, the Secretary of Commerce and the Director of the National Bureau of Standards.

(6) RESOLUTION ON THE ANTARCTICA SEISMOGRAPH STATION

WHEREAS, The British Antarctica Expedition has continued the operation of the seismograph station established by the Ronne Expedition, and

WHEREAS, This station is of great value to Seismology, therefore be it

RESOLVED, That the American Geophysical Union express its thanks to the Expedition for the operation of the station and express its desire that the interpretation of the seismograms be distributed immediately through the Communication channels established by the Ronne Expedition, and be it further

RESOLVED, That a copy of this resolution be sent to the Leader of the British Antarctica Expedition.

(7) RESOLUTION ON NAVY SEISMOGRAPH STATIONS

WHEREAS, The Navy has established special seismograph stations in the Caribbean and Pacific areas, and

WHEREAS, These stations record many strong earthquakes, and

WHEREAS, These records would be of value in locating earthquakes, therefore be it

RESOLVED, That the American Geophysical Union request the Navy Microseismic Stations to reestablish their program for distributing the interpretation of strong earthquakes to interested seismologists, and be it further

RESOLVED, That a copy of this resolution be sent to the Chief of Naval Operations.

Tellers--Chairman Earl Ingerson reported concerning the election by mail ballot conducted during March 1950. The Committee met on April 12, 1950, tabulated the ballots as follows (the reports of Section elections were presented before the respective sections, but are combined here for ready reference):

Total number of ballots received		1857
Rejected ballots		37
Ballots without identification on outer envelope	13	
Ballots from non-citizens of the United States	11	
Ballots mailed and received late	9	
Ballots from members in arrears in dues	4	
Qualified ballots tabulated		1820

The following officers were elected for the triennium July 1, 1950 to June 30, 1953.

Officers of the Union

President	Walter H. Bucher
Vice President	Francis W. Reichelderfer
General Secretary	K. Hilding Beij

Section of Geodesy

President	Harry H. Hess
Vice President	Albert J. Hoskinson
Secretary	Frank L. Culley

Section of Oceanography

President	A. C. Redfield
Vice President	Roger R. Revelle
Secretary	R. H. Fleming

Section of Seismology

President	Perry Byerly
Vice President	Leonard M. Murphy
Secretary	James T. Wilson

Section of Volcanology

President	Esper S. Larsen, Jr.
Vice President	Thomas P. Thayer
Secretary	Felix Chayes

Section of Meteorology

President	Benjamin Holzman
Vice President	Hurd C. Willett
Secretary	Woodrow C. Jacobs

Section of Hydrology

President	F. J. Veihmeyer
Vice President	Merrill Bernard
Secretary	Ray K. Linsley, Jr.

Section of Terrestrial Magnetism and Electricity

President	J. Wallace Joyce
Vice President	Victor Vacquier
Secretary	Leroy R. Alldredge

Section of Tectonophysics

President	Robert Balk
Vice President	Francis Birch
Secretary	Wilbur S. Burbank

[The Special Committee of Tellers was appointed by President Bucher after consultation with Section officers, as follows: Earl Ingerson (Chairman); Gunnar Leifson and E. J. Parkin (Geodesy); Benjamin L. Snavely (Seismology); Robert D. Fletcher and Philip Clapp (Meteorology); H. F. Johnston and J. R. Balsley (Terrestrial Magnetism and Electricity); J. H. Hawley, Harry S. Ladd, Boyd E. Olson, and E. O. Hulbert (Oceanography); Herbert E. Hawkes and Robert R. Coats (Volcanology); G. Earl Harbeck, Jr. and Max A. Kohler (Hydrology); L. H. Adams and Harold M. Bannerman (Tectonophysics). While the elected candidates only are presented, any member may have the detailed results for his confidential use upon application.]

APPENDIX A--MINUTES OF THE ANNUAL MEETING OF 1950, SECTION OF GEODESY, AMERICAN GEOPHYSICAL UNION

The Thirty-First Annual Meeting of the Section of Geodesy was called to order by President H. W. Hemple at $14^{h}00^{m}$, May 1, 1950, in the Lecture Room, National Academy of Sciences, Washington, D. C. About 130 members and guests were in attendance.

The minutes of the Thirtieth Annual Meeting were declared approved as published in the Transactions [v. 30, pp. 466-467, 1949].

J. B. Small, the Section's Membership Committeeman, reported on the continued increase of membership of the American Geophysical Union.

N. F. Braaten, Editor for the Section of Geodesy, reported a backlog of papers were awaiting publication and that several papers presented at the previous annual meeting were yet to be published.

R. M. Wilson, Chairman of the Resolutions Committee, reported no resolutions had been offered.

After announcement of the results of the recent election by Gunnar Leifson of the Committee of Tellers [see preceding page], and after routine announcements, scientific papers were presented as shown on the program for the Section of Geodesy on page 317 of the Transactions for 1950. The abstracts of most of these are presented on pages 323-324.

Walter D. Lambert made an announcement of the tentative plans for the next assembly of the International Union of Geodesy and Geophysics to be held at the University of Brussels in the modern part of the City of Brussels, August 21 to September 1, 1951.

The meeting adjourned at $16^{h}50^{m}$.

FRANK L. CULLEY, Secretary

APPENDIX B--MINUTES OF THE ANNUAL MEETING OF 1950, SECTION OF SEISMOLOGY, AMERICAN GEOPHYSICAL UNION

The Thirty-First Annual Meeting of the Section of Seismology was held Tuesday, May 2, 1950 in the Auditorium of the National Academy of Sciences, Washington, D. C. The meeting was called to order by President Frank Neumann at $09^{h}00^{m}$. The minutes of the last meeting were approved as published in the Transactions [v. 30, pp. 467-468, 1949].

H. Freeborn Johnston of the Committee of Tellers reported the outcome of the elections for the triennium beginning July 1, 1950: President, Perry Byerly; Vice President, L. M. Murphy; and Secretary, James T. Wilson.

The Committee on Resolutions (Leonard M. Murphy, Chairman; Beno Gutenberg; and Daniel T. O'Connell) presented four resolutions for action by the Section. These were approved, and forwarded to the Union for action [see Union Resolutions 4-7, inclusive].

As there was no other business, the Business Meeting was adjourned and was immediately followed by the schedule of scientific papers shown on the program [see Trans., v. 31, p. 317, 1950].

The 1950 Meeting of the Section of Seismology was adjourned at 13^h00^m. About 150 were in attendance, and those present were of the opinion that the meeting was most pleasant and profitable.

FLORENCE ROBERTSON, Secretary

APPENDIX C--MINUTES OF THE ANNUAL MEETING OF 1950, SECTION OF METEOROLOGY, AMERICAN GEOPHYSICAL UNION

The Thirty-First Annual Meeting of the Section of Meteorology was held May 1, 1950, in the Auditorium of the National Academy of Sciences and the General Services Building, and on May 2, 1950, in the Interdepartmental Auditorium. Approximately 350 members and guests attended the meetings of the Section and, in the cases of two of the sessions, the meeting rooms were not large enough to accomodate all those who desired to hear the papers.

The opening business session was held from 09^h00^m to 09^h15^m, May 1, 1950, with President Horace Byers presiding. Since no corrections were received, the President declared the minutes of the Thirtieth Annual Meeting of the Section of Meteorology approved as published in the Transactions [v. 30, p. 468, June 1949]. The President then appointed a Committee on Resolutions (Sidney Teweles, Jr., Chairman, and Werner A. Baum).

The report of the Committee on Extended Forecasting was presented by the Chairman, Jerome Namias. A brief report was given by the Meteorological Editor, Philip F. Clapp who was also a representative of the Tellers and reported on the 1950 elections of officers for the Section of Meteorology [see pp. 487-488].

The opening scientific session was held from 09^h15^m to 12^h00^m, May 1, 1950, under the chairmanship of President Byers and consisted of a Symposium of Atmospheric Physics.

The second session, meeting from 14^h00^m to 17^h00^m, May 1, 1950, was under the chairmanship of H. G. Houghton and consisted of a Symposium of Meteorological Measurements and Instruments.

The third session, meeting from 09^h15^m to 12^h00^m, May 2, 1950, was under the chairmanship of R. D. Fletcher and consisted of a Symposium of Hydrometeorological Problems held jointly with the Section of Hydrology and the American Meteorological Society.

The scientific program for the three sessions consisted of the papers shown on the program, pages 317-138 (abstracts, pp. 325-328) with the following changes:

(10) Read by Florence Robertson.

The following abstract has been received since the program was printed:

(2) W. S. Wilson (Geophysical Institute, University of Alaska, College, Alaska) Surface ozone determinations at College, Alaska--During recent winter months, a series of measurements of surface ozone have been made by the chemical method of oxidation of a solution of sodium arsenite in potassium iodide. The results of these measurements,,which are yet in the developmental stage, show concentrations of ozone of from 2 to 17 mg in 100 cu m of air. Measurements during January, 1950, gave a higher average value than those during February, 1950. The amount of surface ozone showed a rough correlation with temperature; that is, during the early part of a warmer period the quantity of ozone increased.

WOODROW C. JACOBS, Secretary

APPENDIX D--MINUTES OF THE ANNUAL MEETING OF 1950, SECTION OF TERRESTRIAL MAGNETISM AND ELECTRICITY, AMERICAN GEOPHYSICAL UNION

The Thirty-First Annual Meeting of the Section of Terrestrial Magnetism and Electricity of the American Geophysical Union was held on May 2 and 3, 1950 in the Lecture Room of the National Academy of Sciences, Washington, D. C. The first session was called to order at 14^h05^m on Tuesday, May 2, with President Irwin Roman occupying the chair. There were about 150 members and guests in attendance. The minutes of the previous meeting were approved as published in the Transactions [v. 30, p. 469, June 1949].

After routine announcements, the President called for a report of the Nominating Committee appointed in preparation for the 1950 triennial election of officers. The composition of the Committee was as follows: O. W. Torreson, Chairman; R. R. Bodle; C. W. Gartlein; L. H. Rumbaugh; and E. J. Workman. O. W. Torreson reported that the Committee had functioned by meetings and correspondence, and had filed nominations of one candidate for the office of President, tnree for Vice President, and two for Secretary. These nominees were the only ones whose names appeared on the ballot, there being no nominations by petition.

The election returns were summarized by H. F. Johnston, who had served on the Committee of Tellers. He announced that the successful candidates for the three-year term beginning on July 1, 1950 were: J. Wallace Joyce, President; Victor Vacquier, Vice President; and Leroy R. Alldredge, Secretary. Detailed returns will be supplied to members upon inquiry. The President declared the successful candidates elected, and called upon them to rise and receive the congratulations of the assembly.

The Secretary presented a recommendation of the Executive Committee that a Brussels Liaison Committee be established for the purpose of channeling and screening communications concerning the subjects assigned to the various committees of the International Association of Terrestrial Magnetism and Electricity, and other communications concerning the forthcoming Brussels Assembly. A motion calling for the establishment of such a Committee was adopted.

With the conclusion of business the scientific program was begun in accordance with the published schedule as amended [see Transactions, v. 31, pp. 318-319 and 328-330, April 1950] and included the presentation of papers (1) to (7), (7a), and (7b). Paper (8) was read by title, and the session was concluded at 17^h05^m.

The second session was convened at 09^h08^m on Wednesday, May 3. At this session papers (9) to (14) and (14a) were presented.

The three papers not scheduled in the printed program were described in a supplementary bulletin as follows:

(7a) E. O. Schonstedt and H. R. Irons (Naval Ordnance Laboratory, White Oak, Silver Spring, Maryland): Initial test results of the airborne magnetometer for determining all magnetic components.

Abstract--The magnetic airborne detector which was used to detect submarines during World War II has been modified so that the entire magnetic vector can now be recorded. Preliminary flight tests with this instrument between Dayton, Ohio and Key West, Florida and between Dayton, Ohio and Washington, D. C. indicate that it is suitable for use in world-wide magnetic surveys. This instrument will permit world-wide magnetic charts, for which no data have been collected over open ocean areas since 1929, to be brought up to date with precision and economy.

(7b) Elwood Maple, W. A. Bowen (Naval Ordnance Laboratory, White Oak, Silver Spring, Maryland), and S. F. Singer (Applied Physics Laboratory, Johns Hopkins University, Silver Spring Maryland): Rocket measurement of magnetic fields of ionosphere currents near the magnetic equator off the coast of Peru.

Abstract--Direct experimental evidence has been obtained of the existence of ionospheric current sheets. A total-field magnetometer has been used in an Aerobee rocket to measure the Earth's magnetic field to an altitude of approximately 66 mi. The resulting magnetic record indicated a decrease in total field of about four milligauss in an altitude range of from 300,000 ft to 350,000 ft, above the decrease which would be predicted for the main field of the Earth.

(14a) T. N. Gautier and R. W. Knecht (Central Radio Propagation Laboratory, National Bureau of Standards, Washington, D. C.): Formation of a lunar layer in the F-region at Huancayo.

During both sessions, there was active informal discussion from the floor following the presentation of each paper. The meeting was adjourned at $11^{h}20^{m}$.

DAVID G. KNAPP, Secretary

APPENDIX E--MINUTES OF THE ANNUAL MEETING OF 1950, SECTION OF OCEANOGRAPHY, AMERICAN GEOPHYSICAL UNION

The Thirty-First Annual Meeting of the Section of Oceanography was held May 2, 1950, in the Auditorium of the General Services Building, Washington, D. C. The session convened at $09^{h}30^{m}$ and adjourned at $12^{h}30^{m}$. In the absence of the President, George F. McEwen, the Vice President, Edward H. Smith, presided.

As no corrections were offered to the minutes of the previous annual meeting, published in the Transactions [v. 30, p. 470, June 1949], they were considered as approved.

A representative of the Committee of Tellers reported on the outcome of the election of officers of the Section of Oceanography for the triennium beginning July 1, 1950. The following were declared elected: President, A. C. Redfield; Vice President, R. R. Revelle; Secretary, R. H. Fleming.

The Executive Committee of the Section was formally authorized to coordinate plans for participation of the United States oceanographers in the Ninth General Assembly of the International Union of Geodesy and Geophysics to be held in Brussels August 21 to September 1, 1951.

The President of the Section was authorized to appoint members of the Section to assist H. H. Hess and K. O. Emery in their work in connection with the International Committee on Nomenclature of Ocean Bottom Features of the Association of Physical Oceanography (IUGG).

The scientific program at the first session consisted of presentation and discussion of the papers listed in the Program [Transactions, v. 31, p. 319, April 1950] as corrected and amended.

A second scientific session was held in the Auditorium of the National Acacemy of Sciences from $09^{h}15^{m}$ to $12^{h}30^{m}$ on May 3, 1950, that included items listed in the Program but with the following changes:

Paper (9) was presented as the first paper of the second session instead of the last paper of the first session.

Paper (10) by Howard J. Pincus was read by title.

Paper (15) was presented by R. F. A. Studds, Director-Designate of the U. S. Coast and Geodetic Survey.

Two additional papers were read by title: J. D. Isaacs, E. A. Williams, and Carl Eckart: Total reflection of surface waves by deep water; and J. D. Isaacs and T. Rush, Jr.: A strain gage wave meter.

R. H. FLEMING, Secretary

APPENDIX F--MINUTES OF THE ANNUAL MEETING OF 1950, SECTION OF VOLCANOLOGY, AMERICAN GEOPHYSICAL UNION

The Annual Meeting of the Section of Volcanology was called to order by President Wilbur S. Burbank May 1, 1950, at $09^{h}10^{m}$ in the Lecture Room of the National Academy Sciences Building, Washington, D. C.

Robert R. Coats of the Committee of Tellers (who with Herbert E. Hawkes, Jr. represented the Section of Volcanology) reported the election of officers of the Section for the triennium July 1, 1950 to June 30, 1953 as follows: Esper S. Larsen, Jr., President; Thomas P. Thayer, Vice President; and Felix Chayes, Secretary.

The President reported to the Committee for formation of a committee to be called the United States Committee for the Study of Parícutin Volcano. This committee has been established to take the place of a committee by the same name which had been formed under the auspices of the

Division of Geology and Geography of the National Research Council. The old committee was established in July 1944 and will terminate on June 30, 1950.

The President reported that the Executive Committee of the Section of Volcanology has been pleased to appoint the following members to the new committee: Francis G. Wells, Chairman; William F. Foshag, Richard E. Fuller, Earl Ingerson, W. D. Johnston, George E. Kennedy, Robert L. Smith, Howel Williams, and Hatten S. Yoder, who were recommended by Richard E. Fuller, the Chairman of the old committee. The President asked if there were any comments from the floor. There were none.

The meeting adjourned at 12^h05^m.

FRANCIS G. WELLS, Secretary

APPENDIX G--MINUTES OF THE ANNUAL MEETING OF 1950, SECTION OF HYDROLOGY, AMERICAN GEOPHYSICAL UNION

President Straub called the meeting to order at 09^h10^m, May 1, 1950. The Secretary announced the appointment of the following to coordinate planning for the Brussels Assembly of the International Association of Scientific Hydrology in 1951: C. B. Brown, Erosion and Sedimentation; W. O. Field, Snow and Ice; A. N. Sayre, Ground Water; and J.V.B. Wells, Surface Water. Members of the section were urged to submit titles of papers to the appropriate coordinators as soon as possible.

The Secretary announced that the Committee on Awards had determined that no paper submitted during 1949 was eligible for award as the best first paper of the year. The Committee had not been able to complete their selection of the best paper for 1949, and this will be announced in the Transactions at a later date.

M. A. Kohler reported for the Committee of Tellers that the following officers were elected within the Section for the 1950-1953 triennium: President, F. J. Veihmeyer; Vice President, Merrill Bernard; Secretary, Ray K. Linsley.

There being no further business, the President began the technical session, which conformed to the published program except as follows:

(R1) Presented by title.
(R3) R. A. WORK: Report of the Research Committee on Snow (by title)
(R4) Presented by title
(R5) Presented by title
(R6) Presented by title
(R7) Presented by title

The session recessed at 12^h05^m.

Chairman G. A. Hathaway called the second session to order at 14^h10^m. The printed program was followed with the exception of:

(R9) Presented by T. G. McLaughlin
(17) Presented by title
(18) Presented by C. E. Jacob
(R12) Presented by W. B. Langbein

The session recessed at 16^h50^m.

The third session was held jointly with the Section of Meteorology and the American Meteorological Society with R. D. Fletcher as Chairman. It is reported under the minutes of the Section of Meteorology.

Chairman W. B. Langbein called the final session to order at 14^h00^m on May 2, 1950. The published program was followed without exception. The members of the panel on "Hydrology in arid regions" were Peveril Meigs, C W. Thornthwaite, A. N. Sayre, Guy Stewart, and Malcolm Rigby in addition to Merrill Bernard who served as moderator. The meeting was adjourned at 16^h45^m.

RAY K. LINSLEY, Secretary

APPENDIX H--MINUTES OF THE ANNUAL MEETING OF 1950, SECTION OF TECTONOPHYSICS, AMERICAN GEOPHYSICAL UNION

The first session of the Annual Meeting of the Section of Tectonophysics of the American Geophysical Union was held on May 2, 1950, in the Lecture Room of the National Academy of Sciences, Washington, D. C. The meeting was called to order by President M. King Hubbert at 09^h10^m.

The minutes of the previous meeting were approved as published in the Transactions [v. 30, p. 472, June 1949]. L. H. Adams presented the report of the Committee of Tellers for the Section and announced the election of the following officers of the Section of Tectonophysics for the three-year period beginning June 30, 1950: Robert Balk, President; Francis Birch, Vice President; and Wilbur S. Burbank, Secretary.

The business meeting was adjourned and the presentation of papers of the first scientific session followed.

Papers (1) to (7) of the program given on pp. 337-338 of the Transactions were presented, and the session adjourned at 12^h35^m.

The second scientific session was held on May 3, in the Auditorium of the General Services Administration Building, and was called to order by President Hubbert at 09^h10^m. Papers (8) to (12) and (14) of the program were presented. Paper (13) was read by title, the author having sent his regrets on being unable to attend. The meeting was adjourned at 12^h00^m.

WILBUR S. BURBANK, Secretary

SPECIAL ANNOUNCEMENTS

Dates set for Thirty-Second Annual Meeting--The Thirty-Second Annual Meeting will be held in Washington, D. C., April 30 to May 2, 1951 (Monday to Wednesday). Begin making your plans now to attend.

Results of elections--The results of the elections of officers for the triennium July 1, 1950 to June 30, 1953 are set forth on pp. 487-488 of this issue of the Transactions.

Ninth General Assembly of the International Union of Geodesy and Geophysics--Plans for the Ninth General Assembly of the IUGG, which will be held in Brussels, Belgium, from August 21 to September 1, 1951, are progressing well. Dr. A. Letroye, Vice President of the Belgian Organizing Committee, has recently been in the United States and outlined plans developed so far. The meetings will be held at the University of Brussels, somewhat outlying in a new quarter of the city, rebuilt after World War I. Hotels and student homes in the vicinity will provide lodging for those attending, but each person must arrange for his own reservations. A number of excursions is being planned - to Antwerp, Namur, Ghent, Liege, Bruges, and elsewhere - but none outside of Belgium is being contemplated. Facilities will be provided for a headquarters office for the IUGG and each of the Associations, and a local Secretary is being arranged for each of these. It is anticipated that the formal invitation will be forwarded through diplomatic channels in October.

Those who plan to attend should make travel reservations very soon, especially those going by boat. The office of AGU should also be advised. A list of the hotels and other lodging accommodations as well as other information will be supplied to those who so advise, as soon as provided by the Belgian Organizing Committee. Those who plan to present papers should forward word of these, with title and abstract, to the office of AGU as promptly as possible, and papers should be forwarded well in advance of the meeting.

The Secretary of the Belgian Organizing Committee is Dr. C. Charlier, Observatoire d'Uccle, Anvenue Circulaire No. 3, Uccle, Belgique.

Watch the Special Announcements for further word of this meeting.

Civil Service examination for Oceanographers--As we go to press, the U. S. Civil Service Commission has just announced an unassembled examination for Oceanographers for positions located mainly in the U. S. Navy Hydrographic Office and the U. S. Coast and Geodetic Survey, positions ranging from GS-7 through GS-15 at salaries from $3825 to $10,000 per year. Get into touch with your postmaster or the national or regional office of the Civil Service Commission for further details.

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